Kaonic Hydrogen and $K^- p$ Scattering

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Chiral SU(3) effective field theory in combination with a relativistic coupled-channels approach is used to perform a novel analysis of the strong-interaction shift and width in kaonic hydrogen in view of the new accurate DEAR measurements. Questions of consistency with previous $K^- p$ data are examined. Coulomb and isospin breaking effects turn out to be important and are both taken into account in this work.

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The low-energy $\bar{K}N$ system is of special interest as a testing ground for chiral SU(3) symmetry in QCD and, in particular, for the role of explicit symmetry breaking induced by the relatively large mass of the strange quark. Most significantly, the existence of the $\Lambda(1405)$ resonance just 25 MeV below the $K^- p$ threshold makes chiral perturbation theory inapplicable in this channel. Nonperturbative coupled-channels techniques based on driving terms of the chiral SU(3) effective Lagrangian have proved useful and successful in dealing with this problem by generating the $\Lambda(1405)$ dynamically as an $I = 0$ $\bar{K}N$ quasibound state and as a resonance in the $\pi\Sigma$ channel. High-precision $K^- p$ threshold data set important constraints for such theoretical approaches. Now that new accurate results for the strong-interaction shift and width of kaonic hydrogen from the DEAR experiment [1] are available, there is renewed interest in an improved analysis of these data together with existing information on $K^- p$ scattering, the $\pi\Sigma$ mass spectrum, and $K^- p$ threshold decay ratios.

The combination of chiral SU(3) effective field theory with coupled channels was first introduced in Ref. [2] and subsequently further developed and applied to a variety of meson-baryon scattering and photoproduction processes [3–8]. The starting point of this coupled-channels approach is the chiral effective Lagrangian which incorporates the same symmetries and symmetry breaking patterns as QCD and describes the coupling of the pseudoscalar meson octet (π, K, η) to the ground state baryon octet $(N, \Lambda, \Sigma, \Xi)$:

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_{\phi B}. \tag{1}$$

The purely mesonic part of the Lagrangian $\mathcal{L}$ up to second chiral order is given by $\mathcal{L}_\phi$ [9], while the second part $\mathcal{L}_{\phi B}$ describes the meson-baryon interactions and reads at lowest order [10]

$$\mathcal{L}_{\phi B}^{(1)} = i\langle \bar{B}\gamma_\mu [D^\mu, B]\rangle - M_0\langle \bar{B}B\rangle - \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle - \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle, \tag{2}$$

with $\langle\ldots\rangle$ denoting the trace in flavor space. The pseudoscalar meson octet ϕ is summarized in $u_\mu = iu^\dagger\partial_\mu U u^\dagger$ where $U = u^2 = \exp(\sqrt{2}i\phi/f)$, and f is the pseudoscalar decay constant in the chiral limit. The ground state baryon octet is collected in the 3×3 matrix B, M_0 is the common baryon octet mass in the chiral limit, and D, F are the axial vector couplings of the baryons to the mesons. The values of D and F are extracted from the empirical semileptonic hyperon decays. A fit to data gives $D = 0.80 \pm 0.01$, $F = 0.46 \pm 0.01$ [11]. Finally, the covariant derivative of the baryon fields is

$$[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \tag{3}$$

with the chiral connection

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u]. \tag{4}$$

At next-to-leading order the terms relevant for s-wave meson-baryon scattering are

$$\mathcal{L}_{\phi B}^{(2)} = b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + d_1\langle \bar{B}\{u_\mu, [u^\mu, B]\}\rangle + d_2\langle \bar{B}[u_\mu, [u^\mu, B]]\rangle + d_3\langle \bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle \bar{B}B\rangle\langle u^\mu u_\mu\rangle. \tag{5}$$

Explicit chiral symmetry breaking is induced via the quark mass matrix $\mathcal{M} = \mathrm{diag}\,(m_u, m_d, m_s)$ which enters in the combination $\chi_+ = 2B_0(u^\dagger\mathcal{M}u^\dagger + u\mathcal{M}u)$, with $B_0 = -\langle 0|\bar{q}q|0\rangle/f^2$ representing the order parameter of spontaneously broken chiral symmetry.

In the present work the numerical values for the couplings b_i and d_i have been constrained as in the recent coupled-channels analysis of Ref. [7] which includes η photoproduction on nucleons as a high quality data set. We shall allow for small variations around the central values obtained in that work for the following reason: in the approach of [7] only the contact interactions and the direct Born term for meson-baryon scattering were taken into account whereas, in addition, the crossed Born term is included in the present analysis. We can therefore expect small changes in the numerical determination of the coupling constants from a fit to low-energy hadronic data.

In the current investigation we employ a relativistic chiral unitary approach to the strong $\bar{K}N$ interaction based on coupled channels, which accounts for the important contributions of the nearby $\Lambda(1405)$ resonance. By imposing constraints from unitarity we perform the resummation of the amplitudes obtained from the tree level amplitudes and the loop integrals.

The relativistic tree level amplitudes $V_{bj,ai}(s, \Omega; \sigma, \sigma')$ for the meson-baryon scattering processes $B_a^\sigma \phi_i \to B_b^{\sigma'} \phi_j$ (with spin indices σ, σ') at leading chiral orders are obtained from both the contact interactions and the direct and crossed Born terms derived from the Lagrangian $\mathcal{L}$. Since we are primarily concerned with a narrow center-of-mass energy region around the $\bar{K}N$ threshold, it is sufficient to restrict ourselves to the s-wave (matrix) amplitude $V(s)$ which is given by

$$V(s) = \frac{1}{8\pi} \sum_{\sigma=1}^{2} \int d\Omega V(s, \Omega; \sigma, \sigma), \tag{6}$$

where we have averaged over the spin σ of the baryons and s is the invariant energy squared.

For each partial wave unitarity imposes a restriction on the (inverse) T matrix above the pertinent thresholds

$$\mathrm{Im}\, T^{-1} = -\frac{|\boldsymbol{q}_{\mathrm{cm}}|}{8\pi\sqrt{s}}, \tag{7}$$

with the three-momentum $\boldsymbol{q}_{\mathrm{cm}}$ in the center-of-mass frame of the channel under consideration. Hence the imaginary part of T^{-1} is given by the imaginary part of the basic scalar loop integral $\tilde{G}$ above threshold,

$$\tilde{G}(q^2) = \int \frac{d^d l}{(2\pi)^d} \frac{i}{[(q-l)^2 - M_B^2 + i\epsilon][l^2 - m_\phi^2 + i\epsilon]}, \tag{8}$$

where M_B and m_ϕ are the physical masses of the baryon and the meson, respectively. For the finite part G of $\tilde{G}$, one obtains, e.g., in dimensional regularization:

$$G(q^2) = a(\mu) + \frac{1}{32\pi^2 q^2}\bigg\{q^2\bigg[\ln\bigg(\frac{m_\phi^2}{\mu^2}\bigg) + \ln\bigg(\frac{M_B^2}{\mu^2}\bigg) - 2\bigg] + (m_\phi^2 - M_B^2)\ln\bigg(\frac{m_\phi^2}{M_B^2}\bigg) - 8\sqrt{q^2}|\boldsymbol{q}_{\mathrm{cm}}| \times \mathrm{artanh}\bigg(\frac{2\sqrt{q^2}|\boldsymbol{q}_{\mathrm{cm}}|}{(m_\phi + M_B)^2 - q^2}\bigg)\bigg\}, \tag{9}$$

where μ is the regularization scale. The subtraction constant $a(\mu)$ cancels the scale dependence of the chiral logarithms and simulates higher order contributions with the value of $a(\mu)$ depending on the respective channel; cf. [5].

To the order we are working the inverse of the T matrix is written as

$$T^{-1} = V^{-1} + G \tag{10}$$

which yields

$$T = [1 + V \cdot G]^{-1} V. \tag{11}$$

Equation (11) is understood as a matrix equation in each partial wave. The diagonal matrix G collects the loop integrals in each channel. This amounts to a summation of a bubble chain to all orders in the s channel, equivalent to solving a Bethe-Salpeter equation with V as driving term.

We perform a global χ^2 fit to a large amount of data, including $K^- p$ scattering into coupled $S = -1$ channels, the threshold branching ratios of $K^- p$ into $\pi\Sigma$ and $\pi^0\Lambda$ channels, the $\pi\Sigma$ mass spectrum, and the shift and width of kaonic hydrogen recently measured at DEAR [1]. The resulting values of the subtraction constants $a(\mu)$ at $\mu = 1$ GeV are $a_{\bar{K}N}(\mu) = 0.95 \times 10^{-3}$, $a_{\pi\Lambda}(\mu) = -0.59 \times 10^{-3}$, $a_{\pi\Sigma}(\mu) = 1.80 \times 10^{-3}$, $a_{\eta\Lambda}(\mu) = 2.92 \times 10^{-3}$, $a_{\eta\Sigma}(\mu) = 0.98 \times 10^{-3}$, and $a_{K\Xi}(\mu) = 2.90 \times 10^{-3}$. For the couplings b_i, d_i we find (in units of GeV^{-1}) $b_0 = -0.362$, $b_D = 0.002$, $b_F = -0.128$, and $d_1 = -0.11$, $d_2 = 0.05$, $d_3 = 0.31$, $d_4 = -0.32$. The decay constant in the chiral limit, f, is varied between the physical values of the pion decay constant $F_\pi = 92.4$ MeV and the kaon decay constant, $F_K = 112.7$ MeV, since both pions and kaons are involved in the coupled channels. The present fit yields $f = 103.1$ MeV.

The Coulomb interaction has been shown to yield significant contributions to the elastic $K^- p$ scattering amplitude up to kaon laboratory momenta of 100–150 MeV/c [12]. Close to $K^- p$ threshold the electromagnetic meson-baryon interactions are thus important and should not be neglected as in previous coupled-channels calculations [3–6,8]. We account for these corrections by adding the quantum mechanical Coulomb scattering amplitude to the strong elastic $K^- p$ amplitude, $f^{\mathrm{str}}_{K^- p\to K^- p} = 1/(8\pi\sqrt{s})T^{\mathrm{str}}_{K^- p\to K^- p}$. The total elastic cross section is then obtained by performing the integration over the center-of-mass scattering angle. Since this integral is infrared divergent in the presence of the Coulomb ampli-

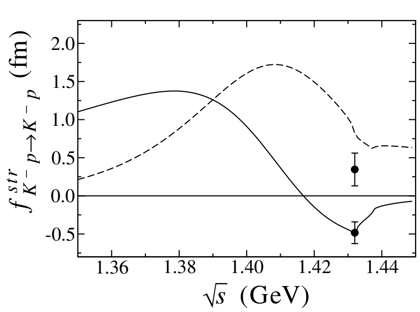

FIG. 1. Real (solid line) and the imaginary part (dashed line) of the strong $K^- p \to K^- p$ amplitude, $f^{\mathrm{str}}_{K^- p\to K^- p}$, as defined in the text. The data points represent the real and imaginary parts of the $K^- p$ scattering length, derived from the DEAR experiment [1] with inclusion of isospin breaking corrections according to Ref. [18].

tude, a cutoff at extreme forward scattering angles must be introduced which we choose to be $\cos\theta_{\rm cm} = 0.966$—the value employed in the data analyses of Refs. [13,14]. The more detailed calculation will be presented in forthcoming work [15].

The results of the fit can be summarized as follows. The strong-interaction part of the $K^- p$ amplitude, $f^{\rm str}_{K^- p \to K^- p}$, is presented in Fig. 1. At threshold we obtain the $K^- p$ strong-interaction scattering length

$$a_{K^- p} = (-0.51 + 0.82i)\ \text{fm}. \tag{12}$$

The $\pi\Sigma$ mass spectrum in the isospin $I = 0$ channel is shown in Fig. 2, while the total cross sections of $K^- p$ scattering to various channels are displayed in Fig. 3.

Additional tight constraints are provided by the well-measured threshold ratios of the $K^- p$ system for which we find:

$$\gamma = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-)}{\Gamma(K^- p \to \pi^- \Sigma^+)} = 2.35,$$
$$R_c = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-, \pi^- \Sigma^+)}{\Gamma(K^- p \to \text{all inelastic channels})} = 0.653, \tag{13}$$
$$R_n = \frac{\Gamma(K^- p \to \pi^0 \Lambda)}{\Gamma(K^- p \to \text{neutral states})} = 0.194.$$

The experimental values $\gamma = 2.36 \pm 0.04$, $R_c = 0.664 \pm 0.011$, $R_n = 0.189 \pm 0.015$ [16,17] are perfectly well reproduced by our approach. (We mention in passing that this fit does not support a pronounced two-pole structure in the region of the $\Lambda(1405)$ as advocated in Ref. [8].)

It turns out, however, that these results cannot be brought to simultaneous satisfactory agreement with the elastic $K^- p$ total cross section and with the strong-interaction shift and width in kaonic hydrogen measured at DEAR [1]. We find $\Delta E = 236$ eV and $\Gamma = 580$ eV (with inclusion of isospin breaking corrections following [18]); see Fig. 4. In comparison with previous coupled-channels calculations, the situation is ameliorated by including electromagnetic corrections to $K^- p$ scattering which are important close to threshold. Nevertheless, inclusion of the Coulomb interaction cannot account for the apparent gap between the DEAR result and the bulk of the existing elastic $K^- p$ scattering data (the latter are, admittedly, of low precision). While there is consistency with the new value for the energy shift in kaonic hydrogen, it is now difficult to accommodate the scattering data with the much improved accuracy of the measured width [1]. (Note that the radiative decays of $K^- p$ into $\Lambda\gamma$ and $\Sigma^0\gamma$ are expected to contribute less than 1% to the decay width of kaonic hydrogen and can be safely omitted [19].)

The results of our calculations represent an "optimal" compromise between the various existing data sets. If, on

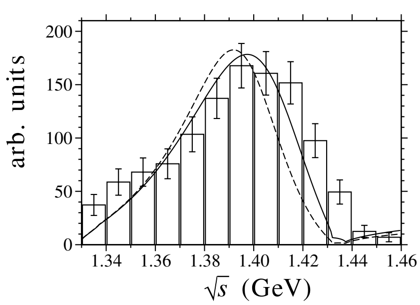

FIG. 2. The $\pi\Sigma$ mass spectrum in the isospin $I = 0$ channel. The solid curve is obtained from the overall χ^2 fit to all available data. The dashed curve is found with the additional constraint of remaining within the error margins of the DEAR data. The experimental histograms are taken from [22]. The statistical errors have been supplemented following [23].

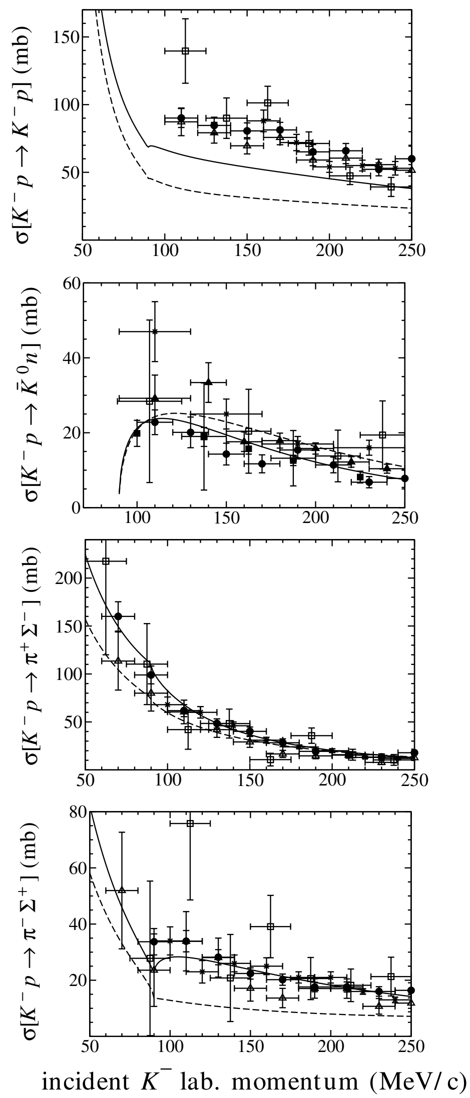

FIG. 3. Cross sections of $K^- p$ scattering into various channels obtained from the overall χ^2 fit to all available data (solid curve) and with the additional constraint of remaining within the DEAR data (dashed line). The data are taken from [13] (empty squares), [14] (empty triangles), [24] (full circles), [25] (full squares), [26] (full triangles), and [27] (stars).

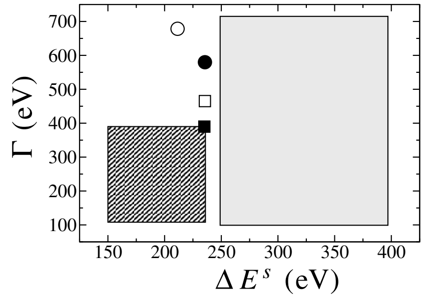

FIG. 4. Results for the strong-interaction shift and width of kaonic hydrogen from our approach, both by using the Deser-Trueman formula [28] (empty circle) and by including isospin breaking corrections [18] (full circle). The DEAR data are represented by the shaded box [1], and the KEK data by the light gray box [20]. The fit restricted to the DEAR data is represented by the small full rectangle (empty rectangle without isospin breaking corrections).

the other hand, one imposes the constraint of remaining strictly within the error band of the DEAR data, the fit yields $\Delta E = 235$ eV, $\Gamma = 390$ eV, which corresponds to a strong-interaction scattering length $a_{K^- p} = (-0.57 + 0.56i)$ fm. With this constraint imposed, we obtain $\gamma = 2.38$, $R_c = 0.631$, $R_n = 0.176$, and a shifted $\pi\Sigma$ mass spectrum (dashed curve in Fig. 2), while the calculated $K^- p$ scattering cross sections move to the dashed curves in Fig. 3.

We have also performed fits omitting the DEAR results. Our calculations are then in good agreement with all scattering data including the elastic $K^- p$ channel. The fits are also within the (larger) error bars of the previous KEK measurement [20]. We have furthermore convinced ourselves that we obtain qualitatively similar results by applying several variants of the approach presented here: first by using only the Weinberg-Tomozawa part of the driving term V, then by adding subsequently the higher order contact interactions b_i, d_i and the direct Born term. These studies will be discussed in detail in a forthcoming report [15].

In conclusion, the present updated analysis of low-energy K^--proton interactions, combining the next-to-leading order chiral SU(3) effective Lagrangian with an improved coupled-channels approach, emphasizes the importance of the constraints set by the new accurate kaonic hydrogen data from the DEAR experiment. At the same time this analysis points to questions of consistency with previously measured sets of $K^- p$ scattering data. Developments aiming for a precision at the level of a few electron volts in the shift and width of kaonic hydrogen, foreseen at DAΦNE in the near future, will further clarify the situation.

The new constrained analysis of the $\bar{K}N$ amplitude presented here is also of considerable interest in view of the continuing quest for kaon condensation in dense baryonic matter, and for the possible existence of deeply bound kaon-nuclear states, an issue which is presently under lively discussion [21].

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