

Comment on “Surprises in Threshold Antikaon-Nucleon Physics”

It has recently been claimed [1] that the DEAR kaonic hydrogen data can be reconciled with $K^- p$ scattering data in a chiral unitary approach. In this Comment we demonstrate that the proposed solution in [1] violates fundamental principles of scattering theory.

Now that new accurate results for the strong-interaction shift and width of kaonic hydrogen from the DEAR experiment are available [2], there is renewed interest in an improved analysis of the $K^- p$ system. In general, non-perturbative coupled-channel techniques based on driving terms from the chiral SU(3) effective Lagrangian have proved successful in the strangeness $S = -1$ sector. However, a thorough investigation [3] of low-energy $K^- p$ interactions within such approaches pointed to questions of consistency of the DEAR experiment with previously measured sets of $K^- p$ scattering data. In contrast, it has been claimed very recently by Oller *et al.* [1] that within a chiral unitary approach both the scattering and the DEAR data can be accommodated. In the following we demonstrate that the proposed solution [1] to this problem violates basic principles of scattering theory and is rather an artifact of the model.

We have first reproduced the A_4^+ fit of [1]. The s -wave interaction kernel of the coupled-channels approach is derived from the Weinberg-Tomozawa term, contact interactions of next-to-leading chiral order as well as direct and crossed Born terms. The s -wave contribution from the crossed Born term, however, leads to unphysical subthreshold cuts which are an artifact of the on-shell formalism used in the coupled-channels approach. These unphysical cuts would not be present in a full field theoretical calculation as discussed in some detail in [3]. The subthreshold singularities induce imaginary pieces in the interaction kernel and hence spoil the exact unitarity of the approach. As a test case we have explicitly checked the S -matrix element for $\pi^0\Lambda \rightarrow \pi^0\Lambda$, the channel with the lowest threshold. Its modulus deviates from 1 in the region below the first inelastic threshold ($\pi\Sigma$). Such unitarity violations can in fact be sizeable.

Nonetheless, we adopt the approach of [1] in order to investigate their results in more detail. A study of the analytical continuation of the T matrix into the complex \sqrt{s} plane reveals an isospin $I = 1$ pole at $\sqrt{s} = (1431 + 1.3i)$ MeV, i.e., right at the $K^- p$ threshold and *above* the physical region. Besides the fact that such a pronounced pole with a width of about 3 MeV on the real axis is not seen empirically (e.g., in the $\pi\Sigma$ invariant mass spectrum), it certainly violates fundamental principles of scattering theory. An unphysical T -matrix pole close to the physical region leads in its vicinity to a strong phase shift variation with energy as shown in Fig. 1. Such a steep decrease of a phase shift is in sharp contradiction to the Wigner condition which—based on a general causality principle for any

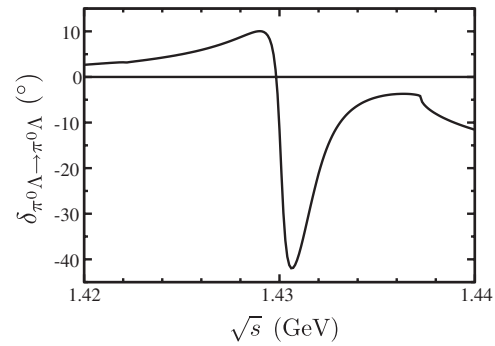


FIG. 1. $\pi^0\Lambda \rightarrow \pi^0\Lambda$ phase shift for the fit A_4^+ of [1].

finite-range interaction—imposes a certain bound on the rate at which the phase shift can change with energy [4]. The limit is given by the range of the underlying interaction, i.e., roughly 1 fm in the case of strong interactions, whereas the steep falloff in Fig. 1 would require an interaction range of more than 300 fm in order to fulfill the Wigner condition. The appearance of the $I = 1$ pole in the upper half plane therefore violates causality and the postulate of maximal analyticity in S -matrix theory.

We conclude by emphasizing that the proposed solution of [1] and the artificial pole arise for certain highly fine-tuned combinations of the effective Lagrangian parameters due to some shortcomings of the applied coupled-channels approach, but should disappear in a full field theoretical calculation. This is further substantiated by the observation that tiny variations in the parameter values of fit A_4^+ lead to sizeable changes in the $K^- p$ scattering length, since the unphysical pole shifts slightly its position, bringing the fit into disagreement with the DEAR measurement.

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