Phases of QCD and PNJL model beyond mean field theory ⋆

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Abstract

We review previous results obtained in the Polyakov loop extended Nambu and Jona-Lasinio model (PNJL) [1, 2, 3] and study corrections to mean field approximation. The presented ansatz is able to cope with the fermion sign problem in the PNJL-model and leads to qualitative improvements even though only of perturbative nature: the expectation value of the Polyakov loop, $\langle \Phi \rangle$, and of its complex conjugate, $\langle \Phi^* \rangle$, are both real quantities that differ from each other once the quark chemical potential μ is non-zero. We conclude that the non-vanishing difference $\langle \Phi^* \rangle - \langle \Phi \rangle$ is an effect beyond mean field caused by fluctuations.

Key words: QCD, PNJL, phase diagram
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1. Introduction

The thermodynamics of QCD has moved to the center of interest as heavy ion collision experiments are able to probe strongly interacting matter at temperatures around $T \approx 200\,\mathrm{MeV}$ and above [4]. Efforts of calculating the thermodynamic properties of QCD in periodic discretized space-time have made tremendous progress. Nevertheless, the results of these lattice QCD calculations will have to be understood in terms of the physical processes involved. It is the evolution of the symmetry properties of QCD with changing temperatures that governs the gross behaviour of hot QCD matter.

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The PNJL model grants insight into a scenario in which spontaneous chiral symmetry breaking appears at similar temperatures as the deconfinement transition which is linked to the spontaneous breakdown of the Z(3) center symmetry of the $\mathrm{SU}(3)_\mathrm{c}$ gauge group of QCD. This Z(3) symmetry breaking at finite temperature is caused by finite expectation values of the Polyakov loop. The astonishing agreement of the PNJL results with lattice QCD calculations [5], especially in detailed quantities such as susceptibilities [2, 3], supports the conjecture that it is these two features implemented in this model that dominate hot QCD around T_c.

2. The PNJL-model

The PNJL-model [2, 3] uses a modified NJL-Hamiltonian $\mathcal{H} = -i\psi^\dagger\,(\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \gamma_4\, m_0 - \phi)\,\psi + \mathcal{V}(\psi,\psi^\dagger)$ [1], where ψ is the $N_f = 2$ doublet quark field and $m_0 = diag(m_u, m_d) = m_0\mathbb{1}$ is the quark mass matrix. In contrast to the NJL case the quarks move in a background colour gauge field, $\phi \equiv A_4/T = iA_0/T$, which is connected to the (traced) Polyakov loop via $\Phi = \frac{1}{N_c}\mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^\beta d\tau A_4\right)\right] = \frac{1}{3}\mathrm{Tr}\, e^{i\,(\phi_3\,\lambda_3 + \phi_8\,\lambda_8)} = \frac{1}{3}\mathrm{Tr}\, e^{i\phi}$. The interaction $\mathcal{V}$ includes pointlike, chiral invariant four-quark couplings, known from the NJL-model.

Gluon dynamics is introduced by a modification of the Euclidean action $\delta S_\mathrm{E} = \frac{V}{T}\mathcal{U}(\Phi)$. The effective potential $\mathcal{U}$ by itself is a Ginzburg-Landau model for the confinement-deconfinement transition in quarkless, pure gauge QCD. Finite values of the Polyakov loop $\langle\Phi\rangle$ break the Z(3) center symmetry of SU(3). The Polyakov loop acts as an order parameter of confinement. The used functional form of $\mathcal{U}$ respects the required Z(3) center symmetry [3]. In addition this form of $\mathcal{U}$ exploits information about the evolution of pure gauge QCD with changing temperature as observed in pure gauge lattice QCD calculations [6a–c].

The NJL parameters of the two flavour model – the current quark mass $m_{u,d}$, the quark coupling strength G and the three momentum cutoff Λ – are chosen such that the model reproduces pion mass, pion decay constant and the chiral condensate [1].

3. Corrections beyond mean field approximation

The introduction of the gluon fields into the NJL-model using the substitution $\mu \to \mu - iA_4$ produces in general formally complex thermodynamic potential. Its imaginary part does not have a direct physical interpretation, so the formal steps leading to this situation have to be reconsidered. For complex potentials the mean field equations have to be modified as they yield twice as many constraints as in the real case. At this point we specify the mean field approximation by minimization of the *real* part of the thermodynamic potential [3]. It is however possible to incorporate the effects of the imaginary part of the action *perturbatively.* To accomplish this, the action is formally separated into a “free” and an “interaction” part. This separation is done such that the “free” action S_0 includes only terms up to second order in the fields, while the “interaction” S_I accommodates all other terms:

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[1] $\boldsymbol{\alpha} = \gamma_0\,\boldsymbol{\gamma}$ and $\gamma_4 = i\gamma_0$ in terms of the standard Dirac γ matrices.

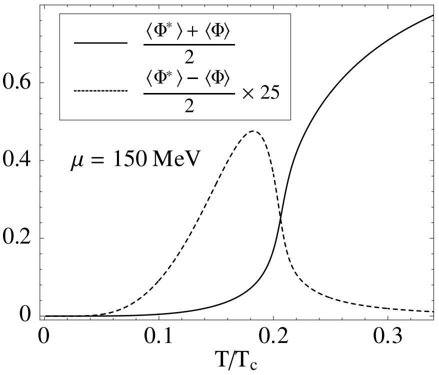

Fig. 1. The sum and difference of the Polyakov loop $\langle\Phi\rangle$ and its complex conjugate $\langle\Phi\rangle^*$ at finite temperature and chemical potential $\mu = 150\,\mathrm{MeV}$.

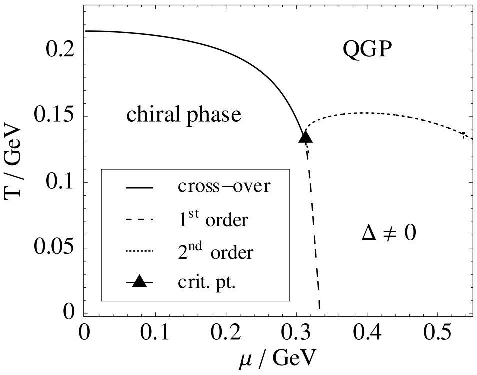

Fig. 2. The phase diagram in the temperature–chemical potential plane including the lowest order perturbative corrections due to fluctuations.

$$S = S_0 + S_\mathrm{I} = \frac{V}{T}\left(\omega_0 + \sum_k {\omega_1}^k\,\xi_k + \frac{1}{2}\sum_{kl}\xi_k\,\omega_2^{kl}\,\xi_l\right) + S_\mathrm{I},$$

where ξ_k are the boson fields shifted by their vacuum expectation values.

The major contribution in this perturbative expansion is expected to originate from non-vanishing derivatives of the potential with respect to the fields, which (by virtue of the self consistency equations) are purely imaginary and may thus be interpreted as source terms. Therefore we order our perturbative expansion in powers of $\delta_l = \sum_k [\omega_2]^{-1}_{lk}\ \mathrm{Im}\left[\omega_1^k\right]$, which is a measure for the interrelation of the steepness of the gaussian potential and the strength of source terms. We only consider the lowest non-vanishing order in the thermodynamic limit, i. e. terms $\propto \delta^0$ and $\propto \delta^1$ at $V \to \infty$.

Already this lowest order correction generates a split of the thermal expectation value of the Polyakov loop $\langle\Phi\rangle$ and its complex conjugate $\langle\Phi\rangle^*$ (see Fig. 1), which is not present at mean field level. Thus this difference is an effect beyond the mean field approach generated by fluctuations of the spatially and temporally constant fields. The gross behaviour of the PNJL thermodynamics is however almost unchanged as can be seen from Fig. 2 in comparison with the results shown in Ref. [3]. An obvious next step will be to study corrections due to pionic fluctuations in a forthcoming effort to incorporate fully dynamical fluctuations.

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