Do ΞΞ bound states exist?

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The existence of baryon-baryon bound states in the strangeness sector is examined in the framework of SU(3) chiral effective field theory. Specifically, the role of SU(3) symmetry breaking contact terms that arise at next-to-leading order in the employed Weinberg power counting scheme is explored. We focus on the $^1S_0$ partial wave and on baryon-baryon channels with maximal isospin since in this case there are only two independent SU(3) symmetry breaking contact terms. At the same time, those are the channels where most of the bound states have been predicted in the past. Utilizing pp phase shifts and $\Sigma^+ p$ cross section data allows us to pin down one of the SU(3) symmetry breaking contact terms and a clear indication for the decrease of attraction when going from the $NN$ system to strangeness $S = -2$ is found, which rules out a bound state for $\Sigma\Sigma$ with isospin $I = 2$. Assuming that the trend observed for $S = 0$ to $S = -2$ is not reversed when going to $\Xi\Xi$ and $\Xi\Xi$ makes also bound states in those systems rather unlikely.

Keywords: Baryon-baryon interactions, chiral effective field theory

I. INTRODUCTION

Dibaryons (as compact six-quark systems or as bound states formed by two conventional octet and/or decuplet baryons) have been intriguing objects of investigations and speculations for many years. While in the purely nucleonic case there is yet again a promising dibaryon candidate\[1\], besides the deuteron, there are indications that the strangeness sector could be specifically rewarding for finding dibaryons\[2\]. Here the by far best-known example is certainly the $H$-dibaryon suggested by Jaffe\[3\], a deeply bound state with quantum numbers of the ΛΛ system, i.e. strangeness $S = -2$, and isospin $I = 0$, and with $J^P = 0^+$. There are also speculations about the existence of other exotic states, notably in the $S = -3$ ($\Omega\Omega$)\[4,5\] and $S = -6$ ($\Omega\Omega$)\[6,7\] systems, see also Refs.\[8,9\].

With regard to two octet baryons, the (approximate) SU(3) flavor symmetry of the strong interaction suggests that bound states could exist for systems with strangeness $S = -3$ and, in particular, $S = -4$\[10\]. Indeed, meson-exchange models like the Nijmegen baryon-baryon ($BB$) interaction\[11,12\], derived under the assumption of (broken) SU(3) symmetry, predict interactions for the $S = -3$ and $-4$ sectors that are fairly strong and attractive and lead to bound states in the $\Sigma\Sigma$ and $\Xi\Xi$ channels\[13,14\]. The situation is somewhat different for $BB$ interactions derived in the constituent quark model by Fujiwara and collaborators\[15\]. While SU(3) flavor symmetry plays likewise a key role in extending the model from the $NN$ and $YN$ interaction (where free parameters are fixed) to the $S = -3$ and $-4$ channels, in this approach it was found that the $BB$ interaction becomes step by step less attractive when going from strangeness $S = 0$ to $S = -4$. In particular, no dibaryon bound states are supported, except for the deuteron. A similar pattern was reported in Ref.\[16\] where the intermediate-range attraction from the scalar-isoscalar (“$\sigma$”) channel was evaluated within a model for correlated $\pi\pi$ and $K\bar{K}$ exchange between octet baryons. Also in this case it was found that the attraction between two baryons, quantified by the effective $\sigma$-meson coupling strength, decreases step by step in the strangeness sector.

Results obtained in lattice QCD calculations are conflicting so far. While a $\Xi\Xi$ bound state was found by the NPLQCD collaboration\[17\] (in the $^1S_0$ state), the HAL QCD collaboration reported only a moderately attractive interaction for that partial wave\[18\].

In the present paper we examine the existence of $\Sigma\Sigma$, $\Xi\Xi$ and $\Xi\Xi$ bound states in the framework of SU(3) chiral effective field theory (EFT). In particular, we explore the role of SU(3) symmetry breaking contact terms that arise at next-to-leading order (NLO) in the perturbative expansion of the baryon-baryon potential. A first study of the baryon-baryon ($BB$) interactions within chiral EFT\[19\] in the Weinberg scheme\[20,21\] for the strangeness $S = -2$, $-3$ and $-4$ sectors was presented in Refs.\[22,23\]. At leading-order (LO) considered in those works the chiral potentials consist of contact terms without derivatives and of one-pseudoscalar-meson exchanges ($\pi$, $K$, $\eta$). Assuming SU(3) flavor symmetry those contact terms and the couplings of the pseudoscalar mesons to the baryons can be related to the corresponding quantities of the $S = -1$ hyperon-nucleon ($YN$) channels. Specifically, the values of the pertinent five low-energy constants (LECs) related to the contact terms could be fixed from the study of the $\Lambda N$ and $\Sigma N$ systems\[18\] and then genuine predictions for the $\Xi\Lambda$, $\Xi\Sigma$, and $\Xi\Xi$ interactions could be made at LO. Strong attraction was found in some of the $S = -2$, $-3$ and $-4$ $BB$ channels, and
several bound states were predicted \cite{23}.

Recently, a $YN$ interaction has been derived up to NLO in chiral EFT by the Jülich-Bonn-Munich group \cite{24,25}. At that order contact terms leading to an explicit SU(3) symmetry breaking appear for the first time \cite{24,25} as mentioned above. Since the sparse experimental information on $\Lambda N$ and $\Sigma N$ scattering could be described rather well with using the SU(3) symmetric terms alone, SU(3) symmetry breaking was simply neglected. In other words it was assumed that the LECs associated with those contact terms are zero. Thus, in the actual calculation the SU(3) symmetry is only broken via the employed physical masses of the involved mesons and octet baryons ($N, \Lambda, \Sigma, \Xi$).

On the other hand, it was also found in Ref. \cite{24} that a simultaneous description of the $YN$ data and the nucleon-nucleon ($NN$) phase shifts is not possible on the basis of SU(3) symmetric contact terms. In particular, the strengths needed for reproducing the $pp$ (or $np$) $^1S_0$ phase shifts and the $^3\Sigma^+$ phase section could not be reconciled in a scenario which maintained SU(3) symmetry for the contact terms. This observation is the starting point for the present study, because it can be used to put constraints on the SU(3) symmetry breaking contact terms. In particular, the situation in the $^1S_0$ partial wave and for $BB$ channels with maximal isospin is rather simple and interesting. Here, there are only two independent SU(3) symmetry breaking LECs at NLO for five physical channels, and for three of those five channels bound states have been predicted in the past. The aforementioned $pp$ phase shifts and the $^3\Sigma^+$ phase section allow us to pin down one of the symmetry breaking LECs and provide a clear-cut indication for the decrease of attraction when one goes from the $NN$ system to $S = -2$, so that a bound state for $\Sigma\Sigma$ with isospin $I = 2$ can be practically ruled out. The other LEC cannot be determined at present and several options for its value are discussed. However, already the assumption that the trend one sees for $S = 0$ to $S = -2$ is not reversed when going to $S = -3$ and $S = -4$ makes bound states in the latter systems rather unlikely.

The paper is structured in the following way: In Sect. 2 we provide a basic introduction to our $BB$ interaction derived in chiral EFT. We also discuss the changes that arise in the interaction when the SU(3) symmetry breaking contact terms are taken into account. Selected results for $BB$ systems with strangeness $S = -2$ to $S = -4$ based on SU(3) symmetric contact terms fixed in a fit to $YN$ data are presented in Sect. 3. In Sect. 4 we introduce the SU(3) symmetry breaking contact terms and show the implications for the $^1S_0$ phase shift in the $\Sigma\Sigma$, $\Xi\Sigma$, and $\Xi\Xi$ channels with maximal isospin. The paper ends with a short summary. Some technical information about our calculation is given in Appendix A.
empirical $\Sigma^+ p$ cross section is grossly overestimated \cite{24}. Evidently, SU(3) symmetry breaking in the contact terms has to be taken into account if one wants to describe $NN$ and $\Sigma N$ scattering simultaneously.

The contact terms, including the SU(3) symmetry breaking corrections that arise at NLO, have been worked out explicitly in Ref. \cite{24} for all octet $BB$ channels from strangeness $S = 0$ to $-4$. There are twelve independent SU(3) symmetry breaking LECs in total, see Ref. \cite{25}, of which six occur in the $^1S_0$ partial wave and the other six in the $^3S_1$. It is impossible to determine all of those based on the presently available experimental information in the strangeness $S = -1$ to $-4$ sectors.

The situation is more favorable, however, in the particular case discussed above, namely for the $^1S_0$ partial wave and $BB$ channels with maximal isospin. Here one obtains

\begin{align}
V_{NN}^{(l=1)} &= C_{1}^{G2} + C_{2}^{G2}(p^2 + p'^2) + \frac{1}{4} m^2 K - m^2_{\pi}), \\
V_{\Sigma N}^{(l=3/2)} &= C_{1}^{G2} + C_{2}^{G2}(p^2 + p'^2) + \frac{1}{4} m^2 K - m^2_{\pi}), \\
V_{\Sigma\Sigma}^{(l=2)} &= C_{1}^{G2} + C_{2}^{G2}(p^2 + p'^2), \\
V_{\Xi\Sigma}^{(l=3/2)} &= C_{1}^{G2} + C_{2}^{G2}(p^2 + p'^2) + \frac{1}{4} m^2 K - m^2_{\pi}), \\
V_{\Xi\Xi}^{(l=1)} &= C_{1}^{G2} + C_{2}^{G2}(p^2 + p'^2) + \frac{1}{2} m^2 (m^2_{\pi} - m^2_{\pi}). \tag{3}
\end{align}

for a particular $BB$ channel. Here, $\mu_{B_1B_2}$ is the reduced mass and $k$ is the on-shell momentum, which is defined by $\sqrt{s} = \sqrt{M^2_{B_1} + k^2 + M^2_{B_2} + k'^2}$. Relativistic kinematics is used for relating the laboratory momentum $p_{lab}$ of the baryons to the center-of-mass momentum. In case of $pp$ and $\Sigma^+ p$, where we compare with experiments, the Coulomb interaction is included. This is done via the Vincent-Phatak method \cite{23}. The potentials in the LS equation are cut off with a regulator function, $f_R(\Lambda) = \exp \left[ - (q^4 + p^4) / \Lambda^4 \right]$, in order to remove high-energy components \cite{21}. In Ref. \cite{24} results for cutoff values in the range $\Lambda = 500 - 650$ MeV were shown and we will consider the same range here. The variation of the results with the cutoff can be viewed as a rough estimate for the theoretical uncertainty \cite{30}. A better method to determine the theoretical uncertainty has recently been proposed for the $NN$ sector \cite{31}, but in view of the scarce data in the strangeness sector and given the exploratory character of our study, we stick to the much simpler procedure of varying the cutoff.

Note that for all the systems listed in Eq. (3) there is no coupling to other partial waves or channels. Thus, differences in the reaction thresholds that generate an additional SU(3) symmetry breaking in the scattering amplitude when the LS equation (4) is solved for coupled channels, are absent. This makes those systems especially suited for isolating SU(3) symmetry breaking effects in the potential.

III. RESULTS BASED ON THE LOW ENERGY CONSTANTS OF OUR NLO $NN$ POTENTIAL

In this section we present predictions for the $\Sigma\Sigma$, $\Xi\Sigma$ and $\Xi\Xi$ channels, where SU(3) symmetry is assumed for the contact terms. To be exact, SU(3) symmetry is utilized to relate the LECs for the $S = -2$, $-3$ and $-4$ systems to those determined in the fit to the $\Lambda N$ and $\Sigma N$ data \cite{24}. The symmetry is broken by the used physical masses of the involved mesons and baryons. Note that the meson masses induce an explicit symmetry breaking into the $BB$ potential while the baryon masses enter only in the course of solving the scattering equation, because they appear in the integral equation in form of the re-
TABLE I. $\Sigma\Sigma$, $\Xi\Sigma$ and $\Xi\Xi$ scattering lengths (in fm) in the $^1S_0$ partial wave. Results are given for our LO \cite{19} and NLO \cite{22} interactions based on LECs fitted to the $YN$ data. For comparison some values for the Nijmegen NSC97 potential \cite{13} and a quark model \cite{17} are also included.

<table>
<thead>
<tr>
<th>System</th>
<th>EFT LO</th>
<th>EFT NLO</th>
<th>NSC97a \cite{13}</th>
<th>NSC97f \cite{13}</th>
<th>fss2 \cite{15}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma\Sigma$ ($I = 2$)</td>
<td>550 $\cdots$ 700</td>
<td>500 $\cdots$ 650</td>
<td>10.32</td>
<td>6.98</td>
<td>-85.3</td>
</tr>
<tr>
<td>$a_{\Sigma\Xi}^{I = 1/2}$</td>
<td>-6.2 $\cdots$ -9.3</td>
<td>60.0 $\cdots$ -286.0</td>
<td>-33.5 $\cdots$ -9.07</td>
<td>-7.4 $\cdots$ -13.5</td>
<td>-80.0 $\cdots$ -2.11</td>
</tr>
<tr>
<td>$a_{\Xi\Xi}^{I = 3/2}$</td>
<td>4.28 $\cdots$ -2.74</td>
<td>8.4 $\cdots$ -13.8</td>
<td>4.13</td>
<td>2.32</td>
<td>-4.63</td>
</tr>
<tr>
<td>$a_{\Xi\Xi}^{I = 1}$</td>
<td>3.92 $\cdots$ -2.47</td>
<td>9.7 $\cdots$ -6.5</td>
<td>17.81</td>
<td>2.38</td>
<td>-1.43</td>
</tr>
</tbody>
</table>

In a corresponding investigation with our LO potential it was found that the interaction in some of the $S = -3$ and $-4$ channels is strongly attractive and even bound states were predicted \cite{22, 23}. The same happens also at NLO as one can see from the results for the $^1S_0$ partial wave summarized in Tables I and II. Specifically, in all channels where large scattering lengths were found at LO, they are likewise large at NLO. And, except for $\Xi\Lambda$, the scattering lengths are large and positive – a clear indication for bound states. In case of $\Sigma\Sigma$ the NLO interaction produces a pole very close to the threshold which, depending on the cutoff, corresponds either to a bound state (large positive scattering length) or to a virtual state (large negative scattering length). The actual binding energies of those states are listed in Table III. Comparing the NLO results with the ones at LO one notices that the $\Xi\Xi$ and $\Sigma\Sigma$ binding energies have become somewhat smaller. Indeed in both cases the systems are now only fairly weakly bound. We do not include the Coulomb interaction in the calculation of the strangeness $S = -2$ to $-4$ sectors. (For bound states this would be technically rather complicated within the Vincent-Phatak method employed by us.) It is quite possible that the additional repulsion due to the Coulomb force could even make the $\Sigma\Sigma$ system unbound. In this context we want to point out that it is good to see that the cutoff dependence of the binding energies is strongly reduced at NLO.

Table II contains also the scattering lengths predicted by the Nijmegen NSC97 meson-exchange model \cite{11} and of a $BB$ potential by Fujiwara and collaborators \cite{15} derived in the quark model. The Nijmegen interaction suggests bound states in the $\Sigma\Sigma$, $\Xi\Sigma$ and $\Xi\Xi$ channels, as can be guessed from the large and positive scattering length. The latest version of the Nijmegen potential \cite{12} produces a bound state in the $\Xi\Xi$ channel \cite{14} too. As already mentioned in the Introduction, no bound states were found for the quark-model interaction \cite{15}, though the $\Sigma\Sigma$ interaction is seemingly very close to producing a bound state as indicated by the large negative scattering length.

TABLE II. Binding energies of various $BB$ bound states (in MeV) in the $^1S_0$ partial wave, for our LO \cite{19} and NLO \cite{22} interactions based on LECs fitted to the $YN$ data.

<table>
<thead>
<tr>
<th>System</th>
<th>EFT LO</th>
<th>EFT NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma\Sigma$ ($I = 2$)</td>
<td>550 $\cdots$ 700</td>
<td>500 $\cdots$ 650</td>
</tr>
<tr>
<td>$\Xi\Sigma$ ($I = 3/2$)</td>
<td>-2.23 $\cdots$ -6.18</td>
<td>-0.58 $\cdots$ -0.19</td>
</tr>
<tr>
<td>$\Xi\Xi$ ($I = 1$)</td>
<td>-2.56 $\cdots$ -7.27</td>
<td>-0.40 $\cdots$ -1.00</td>
</tr>
</tbody>
</table>

IV. RESULTS WITH INCLUSION OF SU(3) SYMMETRY BREAKING CONTACT TERMS

For studying the effects of SU(3) symmetry breaking we performed fits to $NN$ and $YN$ data, requiring that $C^{27}$ is the same in line with the power counting where (SU(3) symmetry breaking) corrections to $C^{27}$ arise at NLO but not to $C^{27}$. With regard to $NN$ the fit was performed to the $^1S_0$ $pp$ phase shifts of the GWU analysis \cite{32, 33}. Since the $pp$ interaction is slightly less attractive than the one in $n\bar{n}$, cf. the scattering lengths of $\approx -17$ fm (for the purely hadronic $pp$ interaction) versus $\approx -23.75$ fm, the amount of SU(3) symmetry breaking we need to introduce is also somewhat smaller. The LEC $C^{27}$ was determined in the $pp$ sector and then taken over in the subsequent calculations in the strangeness sector. It turned out that the actual value of $C^{27}$ found in the fits depends only very weakly on the cutoff mass and, therefore, we adopted a single value for all cutoffs.

The $\Sigma^+p$ interaction was fitted to the corresponding $^1S_0$ phase shift predicted by our chiral EFT $YN$ interaction \cite{24}. We could not simply take over the results of Ref. \cite{24} because that interaction is based on a single decay constant $f_0 \approx f_\pi \approx 93$ MeV. Now we want to take into account also the experimentally known differences between $f_\pi$, $f_\rho$, and $f_K$ in the evaluation of the pertinent coupling constants. In the fit we made sure that there is perfect agreement with the results of \cite{24} in the (low-energy) region where $\Sigma^+p$ cross section data are available. In fact, for one cutoff ($\Lambda = 600$ MeV) we even performed a full fit to all $YN$ data considered in \cite{24} in order to check whether the same $\chi^2$ can be achieved –
which was indeed the case.

It turns out that \( C_1^4 \leq 0 \), i.e., one needs more repulsion to fit \( \Sigma^+ p \) \((Y N)\) data than to fit the \( pp \) \(1S_0\) phase shift. The LECs are graphically presented in Fig. 1, while the phase shifts are shown in Fig. 2. For the former we show the sum of the LO contact term \( C^{27}_3 \) and the SU(3) symmetry breaking contribution for each \( BB \) channel, e.g., \( C^{27}_3 = \bar{C}^{27}_3 + \frac{1}{3} C_3^A (m_K^2 - m_N^2) \) for the \( NN \) system, so that one can see how the repulsion effectively increases when going from \( S = 0 \) to \(-2\). The values of the employed LECs are summarized in Table III.

![Fig. 1. Low-energy constants employed in the different \( BB \) channels, for the considered cutoff values \( \Lambda \). Here \( \bar{C}^{27}_3 (NN) = C^{27}_3 + \frac{2}{3} C_5^A (m_K^2 - m_N^2) \), etc., see Eq. (3). \( C^{27}_3 \) is in units of \( 10^4 \) \(\text{GeV}^{-2}\).](image)

TABLE III. Employed low energy constants for various cut-offs. The values for \( C^{27}_3 \) are in \( 10^4 \) \(\text{GeV}^{-2}\), those for \( C^{27}_3 \) and \( C_3^A \) in \( 10^5 \) \(\text{GeV}^{-4}\).

<table>
<thead>
<tr>
<th>( \Lambda ) (MeV)</th>
<th>( C^{27}_3 )</th>
<th>( C^{27}_3 )</th>
<th>( C_3^A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.15196</td>
<td>2.26</td>
<td>-2.6014</td>
</tr>
<tr>
<td>550</td>
<td>0.32963</td>
<td>2.26</td>
<td>-3.8346</td>
</tr>
<tr>
<td>600</td>
<td>0.61394</td>
<td>2.26</td>
<td>-5.7731</td>
</tr>
<tr>
<td>650</td>
<td>1.0752</td>
<td>2.26</td>
<td>-8.8719</td>
</tr>
</tbody>
</table>

The experimental \( \Sigma^+ p \) cross section provides an upper limit on the phase shift for the \( \Sigma^+ p \) \(1S_0\) partial wave. The limit can be derived from the expression for the partial cross section,

\[
\sigma_{\Sigma^+ p, J} = \frac{(2J + 1)\pi}{k^2} \sin^2 \delta_J, \tag{5}
\]

\( J \) being the total angular momentum, by assuming that the \( 1S_0 \) contribution alone already saturates the cross section data. Pertinent results are included in Fig. 2, see the filled circles. Obviously for our EFT interaction [24] (but also for most of the \( YN \) potentials based on meson exchange [11, 12, 34, 35]) the predicted \( 1S_0 \) amplitude is very close to saturating the \( \Sigma^+ p \) cross section alone. The hatched band in Fig. 2 indicates the predictions one would get for the \( \Sigma^+ p \) channel with the LECs fitted to the \( pp \) \(1S_0\) phase shifts. Evidently, the assumption of SU(3) symmetry for the contact terms is in clear contradiction with the experimental information.

Note that there is also a phase shift analysis for \( \Sigma^+ p \) [36] at a single momentum, namely \( p_{lab} = 170 \) MeV/c, which suggests a value of around 26 degrees for the \( 1S_0 \) partial wave. However, that analysis is not model independent and, therefore, we have more confidence in our own results determined by a fit to existing \( YN \) data within chiral EFT.

Once we have determined \( C^{27}_3, C^{27}_3, \) and \( C_3^A \) from our fit to the \( pp \) and \( pp + 1S_0 \) phase shifts, we can make predictions for the \( \Sigma \Sigma \) case, see Eq. (3). Corresponding results are shown in Fig. 3. The phase shifts attest that there is a sizable attraction in this channel but the actual values are in the order of 30 degrees and, thus, far away from the SU(3) symmetric case discussed in Sect. 2 where the \( \Sigma \Sigma \) system with \( J = 2 \) was more or less bound. In particular, the predicted scattering lengths are now around \(-3.2 \) to \(-3.4 \) fm only. Indeed, the present result at NLO that follows directly from the SU(3) symmetry breaking observed between \( pp \) and \( \Sigma^+ p \) practically rules out a bound state in this channel.

The actual value of \( C_2^2 \) can be only determined by a fit to pertinent \((\Xi N)\) and/or \((\Xi \Xi)\) data. Since such data are not available, in the following let us consider some exemplary choices for \( C_2^2 \). In particular, we presume that the magnitude of the SU(3) breaking LEC \( C_2^2 \) is comparable to \( C_1^4 \) and that the trend in the SU(3) symmetry breaking we see for \( NN \Rightarrow \Sigma N \Rightarrow \Sigma \Sigma \) is not reversed for the \( S = -3 \) and \(-4 \) systems. The latter means that we suppose \( C_2^2 \) to be positive, based on its definition via Eq. (3). A simple assumption is \( C_2^2 \approx 0 \), so that there is no further SU(3) symmetry breaking in the contact terms beyond \( S = -2 \). The other extreme consists in assuming that \( C_2^2 \approx -C_1^4 \), which implies that the same SU(3) symmetry breaking required to describe \( pp \) and \( \Sigma^+ p \) occurs also between the \( S = -2, -3, \) and \(-4 \) \( BB \) systems. Finally, we consider an intermediate case, namely \( C_2^2 \approx -C_1^4 / 2 \).

Predictions for the \( \Xi \Xi (I = 3/2) \) and \( \Xi \Xi (I = 1) \) \(1S_0\) phase shifts resulting from the three choices are presented in Fig. 4. In the case \( C_2^2 \approx 0 \) (hatched band) the only SU(3) symmetry breaking effects in the potential (as compared to \( \Sigma \Sigma \)) come from the one- and two-meson exchange contributions. One notices a clear increase in the attraction for \( \Xi \Xi \) and \( \Xi \Xi \) in comparison to the \( \Sigma \Sigma \) results, cf. Fig. 3 with Fig. 4. Specifically, for \( \Xi \Xi \) the phase shifts reach almost 60 degrees, i.e. similar values as in the \( pp \) system. Introducing an explicit SU(3) symmetry breaking in the contact terms leads to the results represented by the filled bands \((C_2^2 \approx -C_1^4 / 2) \) and dotted bands \((C_2^2 \approx -C_1^4) \), respectively. Now the predicted phase shifts for the \( 1S_0 \) partial wave are much smaller and especially in the \( \Xi \Xi \) case the reduction is drastic.
FIG. 2. $pp$ and $\Sigma^+p$ phase shifts in the $^1S_0$ partial wave. The filled band represent our results at NLO. The hatched band shows $\Sigma^+p$ result based on LECs fixed by a fit to $pp$ phase shifts. The $pp$ phase shifts of the GWU analysis [33] are shown by circles. In case of $\Sigma^+p$ the circles indicate upper limits for the phase shifts, deduced from the $\Sigma^+p$ cross section, see text.

FIG. 3. Phase shifts for the $^1S_0$ partial wave in the $\Sigma\Sigma$ channel with isospin $I=2$. The band is our prediction based on the LECs $\tilde C^{27}$ and $\tilde C^{27}$ fixed from a fit to $pp$ and $\Sigma^+p$, see Eq. (49).

The scattering length for the $\Xi\Sigma$ channel are in the range of $-3.7$ to $-2.8$ fm for the choice $C_\Sigma^3 \approx 0$, but reduce to $-1.3$ to $-1.8$ fm for $C_\Sigma^2 \approx -C_\Sigma^2/2$, and to $-0.7$ to $-1.0$ fm for $C_\Sigma^3 \approx -C_\Sigma^3$. For $\Xi\Xi$ we obtain $-7.0$ to $-13.5$ fm, $-1.6$ to $-1.8$ fm, and $\approx 0.7$ fm, respectively.

We have also performed calculations based on the $np$ $^1S_0$ phase shifts as starting point instead of the $pp$ values. In this case there is a somewhat stronger SU(3) symmetry breaking between $np$ and $\Sigma^+p$ and, accordingly, the resulting $\Sigma\Sigma$, $\Xi\Sigma$ and $\Xi\Xi$ phase shifts are then reduced by roughly 10% as compared to the ones presented in Figs. 2 and 3.

There are results for the $\Xi\Xi$ $^1S_0$ partial wave from lattice QCD calculations. The ones reported by the NPLQCD collaboration [17] suggest a bound state with $E_B = -14.0 \pm 1.4 \pm 6.7$ MeV. The calculation was performed for a pion mass of $m_\pi = 389$ MeV and for $M_\Xi = 1349.6$ MeV. In contrast, no bound state was found by the HAL QCD collaboration [18]. In this calculation, that corresponds to $m_\pi = 510$ MeV and $M_\Xi = 1456$ MeV, the interaction in the $^1S_0$ partial wave is only moderately attractive and the phase shifts rise only to a maximum of around $20 \pm 10$ degrees. Interestingly, the EFT predictions based on the choice $C_\chi^2 \approx -C_\chi^2/2$ are fairly close to those results. Our investigations in Refs. [37, 38] suggest that the actual value of the pion mass does not play an important role in the $\Xi\Xi$ system and, therefore, we do not expect sizable changes in the lattice results once calculations for masses closer to the physical value become feasible. The $\Xi$ mass is only marginally larger than the physical mass (which is about 1320 MeV) in case of the NPLQCD collaboration so that it should not distort the results. In any case, a smaller baryon mass would rather lead to a reduction of the attraction than to an enhancement, cf. the discussion below.

Finally, let us comment on the role played by the SU(3)
with a shift from $\Sigma\Sigma$ (Fig. 3) to $\Xi\Xi$ (Fig. 4) in the scenario and increase in the corresponding reduced masses. It is assumed in extrapolating to the strangeness trialials and in our EFT interactions when SU(3) symmetry the same mechanism is also responsible for the $\Xi\Xi$, etc.

The work of Miller for simple potential models. Clearly, states with increasing baryon masses as demonstrated in potentials this has a drastic effect and leads to bound states. His argument is easy to understand in terms of the Schrödinger equation, 

$$-\frac{d^2u}{dr^2} + 2\mu_{B_1}B_2 V_{B_1B_2} u = k^2 u$$

where $u(r)$ is the wave function. If SU(3) symmetry is approximately fulfilled then $V_{NN} \approx V_{\Xi\Xi}$. However, since the physical mass of $\Xi$ is significantly larger than the one of the nucleon, the effective strength of the interaction is increased when it is multiplied with the appropriate reduced mass $\mu_{B_1B_2}$. For example, for $\Xi\Xi$ one has $\mu_{\Xi\Xi}/\mu_{NN} \approx 1.40$, i.e. there is a 40% increase in the effective strength of the interaction as compared to $NN$, while for $\Sigma\Sigma$ one gets $\mu_{\Sigma\Sigma}/\mu_{NN} \approx 1.27$. For attractive potentials this has a drastic effect and leads to bound states with increasing baryon masses as demonstrated in the work of Miller for simple potential models. Clearly, the same mechanism is also responsible for the $\Xi\Xi$, etc. bound states that one observes in meson-exchange potentials and in our EFT interactions when SU(3) symmetry is assumed in extrapolating to the strangeness $S = -3$ and $-4\ BB$ systems. Indeed, the increase in the phase shift from $\Sigma\Sigma$ (Fig. 3 to $\Xi\Xi$ (Fig. 4) in the scenario with $C_{2}^\chi \approx 0$ reported above is primarily dictated by the increase in the corresponding reduced masses.

The actual $pp$ and $\Sigma^+p$ phase shifts suggest that there is no such net increase in the attraction when going to the strangeness sector. Thus, in practice the SU(3) symmetry breaking LEC $C_{1}^\chi$ (more than) compensates effectively the impact of the increase in the reduced mass. Indeed, the stepwise modification of the contact interaction due to the SU(3) symmetry breaking terms that follows from chiral EFT, cf. Eq. (3), is paralleled by a similar stepwise increase in the reduced mass when going from $NN$ to $\Sigma N$ to $\Sigma\Sigma$, say. Thus, since the mass splitting between $\Sigma$ and $\Xi$ is significantly smaller than the one between nucleon and $\Sigma$, $M_\Xi - M_\Sigma \approx 125\ MeV$ versus $M_\Sigma - M_N \approx 254\ MeV$, one could speculate that the magnitude of the “compensating” SU(3) symmetry breaking LEC ($C_{2}^\chi$) is likewise reduced. If that is so, adopting $C_{2}^\chi \approx -C_{1}^\chi/2$ might be a reasonable choice. In any case, we believe that a realistical estimation for $C_{2}^\chi$ might be provided by $-C_{1}^\chi/2 \geq C_{2}^\chi \geq 0$. But it is obvious from our results that for any value $C_{2}^\chi \geq 0$ the bound states that we find in the $\Sigma\Sigma$, $\Sigma\Xi$ and $\Xi\Xi$ systems for interactions with SU(3) symmetric contact terms (cf. the results presented in Sect. 3) disappear.

V. SUMMARY

In the present paper we examined the question whether baryon-baryon bound states in the strangeness sector could exist in the framework of chiral effective field theory. In particular, we explored the role of SU(3) symmetry breaking contact terms that arise at next-to-leading order in the perturbative expansion in the employed Weinberg scheme. We focused on the $^1S_0$ partial wave...
and on baryon-baryon channels with maximal isospin because in this case there are only two independent SU(3) symmetry breaking contact terms and, at the same time, those are the channels where most of the bound states have been predicted in the past. Utilizing pp phase shifts and $\Sigma^+p$ cross section data allowed us to pin down one of the SU(3) symmetry breaking contact terms and a clear indication for the decrease of attraction when going from the $NN$ system to strangeness $S = -2$ is found, which practically rules out a bound state for the $\Sigma\Sigma$ partial wave with isospin $I = 2$. Furthermore, if that trend observed for $S = 0$ to $S = -2$ is not reversed when going to the corresponding $\Xi\Xi$ and $\Xi\Xi$ channels, which we assumed in the present investigation, then also bound states in the latter systems are rather unlikely.

Experiments for $BB$ systems with $S = -3$ or $-4$ are certainly rather challenging. However, it should be feasible to perform $\Xi\Xi$ and $\Xi\Xi$ correlations measurements in heavy-ion collisions at RHIC or at CERN, similar to those for $\Lambda\Lambda$ reported recently [39]. From such data conclusions on the strength of the interaction in those systems could be drawn and possibly even on the existence of dibaryons. $BB$ systems with strangeness $S = -2$ to $-4$ could be also produced in photon induced reactions on the deuteron at JLab as suggested in Ref. [10], or in corresponding $B^-\bar{B}$ induced reactions at J-PARC [40]. As discussed in [41], from such data one could even deduce the scattering lengths for specific $BB$ channels which would then provide a clear signal for the presence (or absence) of bound states. First and foremost, however, it would be good to resolve the discrepancies in the present lattice QCD calculations for the $\Xi\Xi$ system. Hopefully, this can be done soon, because then we could get already a unique and definite answer.

ACKNOWLEDGEMENTS

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Appendix A: Two–pseudoscalar-meson exchange contributions

The spin-momentum part of the interaction in the $\Sigma\Sigma$, $\Xi\Sigma$ and $\Xi\Xi$ channels is the same as in the $YN$ case and is described in detail in the Appendix A of Ref. [24]. There are, however, some changes in the isospin coefficients for $\Sigma\Sigma$ as compared to $\Sigma\Pi$ because the roles of $K$ and $K$ and likewise of $N$ and $\Xi$ are interchanged. For convenience we summarize the isospin factors for the $\Sigma\Sigma$ ($I = 3/2$) case in Table 1. Those for $\Xi\Xi$ are identical to the ones for $NN$, with the replacement $N \leftrightarrow \Xi$ and $K \leftrightarrow \bar{K}$. The isospin factors for $\Sigma\Xi$ and $I = 2$ are given in Table 1. Note that the isospin factors for the one-pseudoscalar-meson exchange can be found in Table 3 of Ref. [23] (for $\Sigma\Xi$) and in Table 3 of Ref. [22] (for $\Sigma\Sigma$).

The explicit SU(3) symmetry breaking in the decay constants is taken into account. The empirical values for these constants are [42]

\begin{align*}
    f_\pi &= 92.4 \text{ MeV}, \\
    f_\eta &= (1.19 \pm 0.01) f_\pi, \\
    f_K &= (1.30 \pm 0.05) f_\pi .
\end{align*}

and we use the central values in our study. A somewhat smaller SU(3) symmetry breaking occurs also in the axial coupling constants, see [43–45] but also [46, 47]. These effects are not taken into account here.

As discussed in Appendix A.1 of Ref. [24] the evaluation of the two-pseudoscalar-meson exchange gives also rise to a polynomial part. We assume here that those contributions only renormalize the LO and NLO contact terms and, therefore, they are not considered. Some of the terms omitted involve the masses of the pseudoscalar mesons and the SU(3) symmetry breaking generated by them is assumed to be absorbed by the SU(3) symmetry breaking contact terms $C_I^0$ and $C_I^2$. In principle, there is also an SU(3) symmetry breaking due to differences in the baryon masses as discussed in Appendix B.2 of Ref. [24]. However, since we consider here only channels with the same baryons in the initial and final states, their effects are tiny and are not taken into account here.

TABLE I. Isospin factors $I$ for $\Xi\Sigma$ with $I = 3/2$ for planar box, crossed box, triangle, and football diagrams consecutively. $B_iB_{ir}$ indicates the two baryons in the intermediate state and $\pi\pi$ etc. the exchanged pair of mesons $M_1M_2$ for planar box and crossed box diagrams. In case of the triangle diagrams there is only a single baryon in the intermediate state. See Ref. [24] for details of notation.

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TABLE II. Isospin factors $I$ for $\Sigma\Sigma$ with $I = 2$ for planar box, crossed box, triangle, and football diagrams consecutively. $B_iB_{ir}$ indicates the two baryons in the intermediate state and $\pi\pi$ etc. the exchanged pair of mesons $M_1M_2$ for planar box and crossed box diagrams. In case of the triangle diagrams there is only a single baryon in the intermediate state. See Ref. [24] for details of notation.

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