Comment about constraints on nanometer-range modifications to gravity from low-energy neutron experiments

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Abstract

A topic of present interest is the application of experimentally observed quantum mechanical levels of ultra-cold neutrons in the earth’s gravitational field for searching short-range modifications to gravity. A constraint on new forces in the nanometer-range published by Nesvizhevsky and Protasov follows from inadequate modelling of the interaction potential of a neutron with a mirror wall. Limits by many orders of magnitude better were already derived long ago from the consistency of experiments on the neutron-electron interaction.

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In a recent experiment, quantum mechanical levels of the neutron in the earth’s gravitational field were observed above a flat neutron optical mirror by measuring the transmission of ultra-cold neutrons through a narrow horizontal channel formed by the mirror and an absorber, as a function of the channel width [1]. The nuclear Fermi potential of the mirror material is given by

\[ U_F = \frac{2\pi\hbar^2}{m_n} Nb, \]  

(1)

where \( m_n \) is the neutron mass, and \( b \) is the coherent neutron scattering length of the mirror nuclei with number density \( N \). The energies of the lowest energy levels of the neutron in the potential well formed by the earth gravitational potential and the mirror are in the peV range whereas a typical value for \( U_F \) is 100 neV. It was pointed out that additional short-range interactions close to the mirror would modify the transmission pattern. The authors of refs. [2, 3] claim that a competitive limit on non-Newtonian interactions in the nm range follows from the absence of a bound state due to the hypothetic short-range interaction. We show that this claim is not valid.

A common parameterisation of short-range modifications of the gravitational interaction employs a Yukawa-type potential between two masses \( m \) and \( M \),

\[ V_G (r) = -G\frac{mM}{r}\alpha_G \exp \left( -r/\lambda \right). \]  

(2)

For the extended mass of a flat mirror, integration of eq. (2) provides the effective potential for a neutron situated at distance \( z \) outside the mirror with mass density \( \rho \),

\[ V_{\text{eff}} (z) = -U_0 \exp \left( -z/\lambda \right), \quad U_0 = 2\pi G\alpha_G m_n \rho \lambda^2. \]  

(3)

The authors of refs. [2, 3] replaced the potential inside the mirror by infinite repulsion and thus consider the potential well \( U = \infty \) for \( z \leq 0 \), and \( U = -U_0 \exp \left( -z/\lambda \right) \) for \( z > 0 \), with \( U_0 > 0 \). The condition for the absence of a bound state in this well,

\[ U_0 m_n \lambda^2 < 0.723 \hbar^2, \quad \alpha_G > 0, \]  

(4)
follows from transformation of the Schrödinger equation for a particle moving in the exponential pocket to Bessel’s differential equation via the substitution \( z = \exp (-x/(2\lambda)) \). From nonobservation of an effect due to a bound state in their experiment [11], they conclude from condition (4) an excluded range of values for the parameter pair \((\alpha_G, \lambda)\). However, their limit is very poor: e.g. for \( \lambda = 10^{-9} \text{ m} \), \( U_0 < 30 \mu \text{eV} \), still allowing for values \( U_0 \gg U_F \). For such large values of \( U_0 \) the assumption of an infinite potential barrier becomes completely unrealistic, since the treatment in refs. [2, 3] neglects the effect of the short-range interaction within the mirror.

Evaluating the integral leading to \( V_{\text{eff}}(z) \) also for values \( z < 0 \), we obtain

\[
U(z) = \begin{cases} 
-U_0 \left( 2 - \exp \left( z/\lambda \right) \right) + U_F & z \leq 0 \\
-U_0 \exp \left( -z/\lambda \right) & z > 0 
\end{cases}.
\] (5)

Deep inside the mirror \(|z| \gg \lambda\) the short-range interaction thus contributes \(-2U_0\) to the neutron optical potential, such that neutrons would no more be totally reflected if \( U_0 \) is larger than half of \( U_F \).

Using the potential given in eq.(5), one may exclude values of the parameter pair \((\alpha_G, \lambda)\) which are orders of magnitude more stringent than those quoted in refs. [2, 3]. A sensitive method consists in the comparison of neutron scattering amplitudes derived from neutron optical and neutron scattering methods, i.e. for momentum transfer \( hq = 0 \) and \( hq \neq 0 \). A contribution to the scattering amplitude of a nucleus with mass \( M \) from a Yukawa-type potential in Born approximation is given by

\[
f_G(q) = -\frac{m_n}{2\pi\hbar^2} \int V_G(r) \exp(iq \cdot r) \, d^3r = \frac{2G\alpha_Gm_n^2M}{\hbar^2} \frac{1}{q^2 + \lambda^{-2}}.
\] (6)

Hence, in a measurement at \( q = 0 \), the Yukawa interaction will contribute to the neutron interaction with a macroscopic body, as given in eq.(5) for a flat body with linear extensions \( \gg \lambda \), whereas in a scattering process with \( q \gg 1/\lambda \) the effect of \( V_G(r) \) will be strongly suppressed due to the \( q \) dependence of \( f_G \). This strategy was employed in measurements of the amplitude \( f_{\text{ne}}(q) = -ZF(q)b_{\text{ne}} \) of neutron-electron scattering, where the known form factor \( F(q) \) of the atomic electron shell with total charge number \( Z \) defines the required \( q \) range for scattering experiments. Koester and coworkers combined measurements of the total scattering cross section for lead and bismuth at energies 1.26 eV and 5.19 eV with neutron optical measurements, from which they derived a value for the neutron-electron scattering length of \( b_{\text{ne}} = - (1.32 \pm 0.04) \times 10^{-3} \text{ fm} \) [3]. An independent determination of \( b_{\text{ne}} \) in measurements of the neutron scattering asymmetry from noble gases with \( q \) values in the range \( 5 \text{ nm}^{-1} < q < 20 \text{ nm}^{-1} \) by Krohn and Ringo provided a value of \( b_{\text{ne}} = - (1.30 \pm 0.03) \times 10^{-3} \text{ fm} \) [3].

Including the contribution due to the hypothetic gravitational short-range interaction and defining \( b_G = -f_G(0) \), the neutron optical potential is given by

\[
U = \frac{2\pi\hbar^2}{m_n} N(b + Zb_{\text{ne}} + b_G).
\] (7)

The cross section for scattering of neutrons with energy \( E_j \) of a few eV is given by (neglecting small corrections)

\[
\sigma(E_j) = 4\pi \left( b + Zf_jb_{\text{ne}} + f_{Gj} \right)^2,
\] (8)

where the number \( f_j \) describes the influence of the atomic form factor and \( f_{Gj} \) originates from integrating the Yukawa-type gravitational interaction over the allowed momentum transfers \( q \), at incident neutron energy \( E_j \). The neutron scattering asymmetry for two values of momentum transfer \( q_1 \) and \( q_2 \) is given by (not discussing recoil)

\[
A = \frac{d\sigma}{d\Omega}(q_1)/d\Omega(q_2) \simeq 1 + 2Z \frac{b_{\text{ne}}}{b} (F(q_1) - F(q_2)) + \frac{2}{b} (f_G(q_2) - f_G(q_1)).
\] (9)
Allowing a contribution of \(b_G\) to \(U\) ranging up to \(Z b_{\text{ne}}\) and focusing on a Yukawa-type interaction with range \(\lambda \geq 1\) nm, we may analyse the situation in leading order in the small amplitudes. For energies \(E_j \gtrsim 1\) eV, we have \(f_{Gj} \ll Z f_j b_{\text{ne}}\) since the values of \(\lambda\) considered are much larger than the size of the atom, and therefore

\[
\sigma(E_j) \simeq 4\pi b (b + 2Z f_j b_{\text{ne}}).
\]

Also the last term in eq. (9) becomes negligible compared to the second one,

\[
A \simeq 1 + 2Z \frac{b_{\text{ne}}}{b} (F(q_1) - F(q_2)).
\]

One thus deals with three observables as functions of the three quantities \(b\), \(b_{\text{ne}}\) and \(b_G\). From this one may obtain a value, respectively, a limit for \(U_0\) for the specified range of \(\lambda\). Indeed such an analysis was already given in 1992 by Leeb and Schmiedmayer [4]. The gravitational short-range amplitude appears to leading order only in \(U\), and the contribution of \(f_G(0)\) should thus appear as a difference between the results for \(b_{\text{ne}}\) obtained by measuring the combination of \(U\) and \(\sigma(E_j)\) [6], and via \(A\) [5]. From the consistency of the experimental results the authors of ref. [4] derived \(b_G = (-1.6 \pm 4.1) \times 10^{-3}\) fm, corresponding to \(U_0 = (-1.4 \pm 3.4) \times 10^{-11}\) eV. In the present parametrisation, this leads, within 90 \% confidence limit, to the constraint

\[
|\alpha_G| \lambda^2 < 530 \text{ m}^2 \quad \text{for } \lambda \geq 1\text{ nm}.
\]

This limit might become somewhat less stringent if one includes two prior determinations of \(b_{\text{ne}}\) obtained for \(^{186}\text{W}\) and bismuth published in [7], which are inconsistent with the results in ref. [6]. On the other hand, a more recent precise determination of \(b_{\text{ne}}\) by neutron transmission through liquid \(^{208}\text{Pb}\) with the neutron time-of-flight method in the neutron energy range 0.08 to 800 eV is in full agreement [8]. In any case the limit stated as competitive in ref. [2] and as new in ref. [3] is by several orders of magnitude worse: \(\alpha_G \lambda^2 < 3.4 \times 10^7 \text{ m}^2 \times (1\text{ nm}/\lambda)^2\) (note that the comparison becomes meaningless for \(\lambda \gtrsim 10\text{ nm}\), where limits more stringent than (12) were already provided by methods not involving free neutrons). Note also that the limit (12) is independent of the sign of \(\alpha_G\), rendering the separate discussion of a repulsive short-range interaction in refs. [2, 3] obsolete.

References