

In-medium chiral $SU(3)$ dynamics and hypernuclear structure

P. Finelli ^a, N. Kaiser ^b, D. Vretenar ^c, W. Weise ^b,

^a*Physics Department, University of Bologna, I-40126 (Italy)*

^b*Physik-Department, Technische Universität München,
D-85747 Garching (Germany)*

^c*Physics Department, University of Zagreb, HR-10002 (Croatia)*

Abstract

A previously introduced relativistic energy density functional, successfully applied to ordinary nuclei, is extended to hypernuclei. The density-dependent mean field and the spin-orbit potential are consistently calculated for a Λ hyperon in the nucleus using the $SU(3)$ extension of in-medium chiral perturbation theory. The leading long range ΛN interaction arises from kaon-exchange and 2π -exchange with Σ hyperon in the intermediate state. Scalar and vector mean fields reflecting in-medium changes of the quark condensates are constrained by QCD sum rules. The model, applied to oxygen as a test case, describes spectroscopic data in good agreement with experiment. In particular, the smallness of the Λ spin-orbit interaction finds a natural explanation in terms of an almost complete cancellation between scalar-vector background contributions and long-range terms generated by two-pion exchange.

Key words: Chiral Dynamics, Hypernuclei, QCD sum rules, Density Functional Theory

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1. *Introduction.* Ever since the discovery of the first Λ hypernucleus in 1953 [1], hypernuclear physics has been an active research area [2,3], in particular once spectroscopic investigations using (K^-, π^-) reactions became available [4]. A most intriguing result has been the extraordinary weakness of the Λ -nucleus spin-orbit interaction. In recent years high quality Λ -hypernuclear spectra produced by (π^+, K^+) reactions confirmed this result. For example, measurements of $E1$ -transitions from p - to s -shell orbitals of a Λ hyperon in ${}^{13}_{\Lambda}\text{C}$ gave a $p_{3/2} - p_{1/2}$ spin-orbit splitting of only (152 ± 90) keV [5] (much smaller than the corresponding 6 MeV in ordinary p -shell nuclei). The same conclusion is

Email address: paolo.finelli@bo.infn.it (P. Finelli).

drawn from an analysis of the excitation energy difference between the 0_1^+ and 2_1^+ states of ${}_{\Lambda}^{16}\text{O}$ [4]. The experimental evidence therefore suggests that the Λ spin-orbit interaction is indeed very weak.

Early theoretical attempts to describe hypernuclear spectra [6] assumed weak couplings between Λ and exchanged bosons in a relativistic mean field model, where “weak” means that the scalar and vector fields experienced by the Λ in the hypernucleus have 1/3 of the strength of the corresponding scalar and vector mean fields for the nucleons. In contrast, a quark model description in which the strange quark inside the Λ does not interact with the up and down quarks of the nucleons, suggests a ratio 2/3 between couplings of the Λ and the nucleons. Subsequently Pirner, Noble and Jennings [7] reconciled the empirical findings with quark model predictions by introducing a strong (negative) $\omega\Lambda\Lambda$ tensor coupling which generates a spin-orbit force with opposite sign so as to yield a small net result. Phenomenological studies along this line [8] have been useful in reproducing the empirical single particle levels for a large set of hypernuclei. However, such a strong (negative) $\omega\Lambda\Lambda$ tensor coupling was considered unnatural.

In recent years new approaches emerged in order to investigate hypernuclear spectra from a microscopic point of view. A connection between quark model features and a relativistic one-boson picture was drawn by the Quark-Meson Coupling (QMC) model [9]. Lenske *et al.* [10] used a density dependent relativistic approach in which the Λ -meson couplings are partly determined from a theoretical ΛN T-matrix and partly fitted to a selected set of data. Despite these constraints, it turned out not to be possible to understand the anomalously small Λ -nucleus spin-orbit force.

The relevance of explicit two-pion exchange contributions to the nuclear force and low-energy observables has become a generally accepted fact. In Refs. [11,12] it has been demonstrated that iterated one-pion exchange and irreducible 2π exchange processes with inclusion of Pauli blocking and Δ intermediate states can generate the correct nuclear binding in the nuclear matter case. For hypernuclei the importance of correlated 2π exchange was pointed out in Ref. [6] which emphasized the role of Σ^* resonances as intermediate states. Only recently [13] the long-range ΛN interaction arising from kaon and 2π exchange, with Σ hyperons as intermediate states and medium insertions, has been explicitly calculated in a controlled expansion in powers of the Fermi momentum k_f .

2. *Λ -hypernuclei in the context of in-medium chiral $SU(3)$ dynamics.* In this work we present an extension to hypernuclei of a relativistic nuclear energy density functional [14,15,16] that combines relevant features of chiral dynamics and the symmetry breaking pattern of low-energy QCD. Chiral pionic fluctuations in combination with Pauli blocking effects [11], Δ excitations and

three-nucleon ($3N$) interactions [12] are superimposed on the condensate background fields and produce the nuclear binding. Scalar and vector mean fields representing the in-medium changes of the quark condensates, which cancel almost completely in their sum, act coherently in their difference to generate the large spin-orbit potential for nucleons in nuclei. This model has been successfully applied [14,15,16] to the description of ground states and collective excited states of open-shell nuclei. In the present work this approach is generalized to study Λ hypernuclei. In particular, we examine whether the novel mechanism for the suppression of the spin-orbit potential proposed in Ref. [13] (and recently also followed in Ref. [17]) at the nuclear matter level works as well in finite hypernuclei.

2.1 *The model.* To describe hypernuclei we generalize the relativistic density functional previously introduced (see Sect. 2.2 in Ref. [16]) adding the hyperon contribution:

$$E_0[\rho] = E_0^N[\rho] + E_0^\Lambda[\rho] , \quad (1)$$

where $E_0^N[\rho]$ describes the core of protons and neutrons (see Eq. (12) in Ref. [16]) and $E_0^\Lambda[\rho]$ is the leading-order term for the single Λ hyperon, decomposed in free and interaction parts:

$$E_0^\Lambda[\rho] = E_{\text{free}}^\Lambda[\rho] + E_{\text{int}}^\Lambda[\rho] , \quad (2)$$

with

$$E_{\text{free}}^\Lambda = \int d^3r \langle \phi_0 | \bar{\psi}_\Lambda [-i\boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + M_\Lambda] \psi_\Lambda | \phi_0 \rangle \quad (3)$$

$$E_{\text{int}}^\Lambda = \int d^3r \left\{ \langle \phi_0 | G_S^\Lambda(\rho) (\bar{\psi}\psi) (\bar{\psi}_\Lambda\psi_\Lambda) | \phi_0 \rangle + \langle \phi_0 | G_V^\Lambda(\rho) (\bar{\psi}\boldsymbol{\gamma}_\mu\psi) (\bar{\psi}_\Lambda\boldsymbol{\gamma}^\mu\psi_\Lambda) | \phi_0 \rangle \right\} . \quad (4)$$

Here $|\phi_0\rangle$ denotes the (hypernuclear) ground state. E_{free}^Λ is the contribution to the energy from the free relativistic hyperon including its rest mass M_Λ . The interaction term E_{int}^Λ includes density dependent hyperon-nucleon vector (G_V^Λ) and scalar (G_S^Λ) couplings. They receive mean-field contributions from in-medium changes of the quark condensates (identified with superscript (0)) and from in-medium pionic fluctuations governed by two-pion exchange processes (with superscript (π)):

$$G_i^\Lambda(\rho) = G_i^{\Lambda(0)} + G_i^{\Lambda(\pi)}(\rho) \quad \text{with} \quad i = S, V . \quad (5)$$

Minimization of the ground-state energy leads to coupled relativistic Kohn-Sham equations for the core nucleons and the single Λ hyperon. Using the

same notation as in Ref. [16] we have:

$$[-i\boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + M_N + \gamma_0 (\Sigma_V + \Sigma_R + \tau_3 \Sigma_{TV}) + \Sigma_S + \tau_3 \Sigma_{TS}] \psi_k = \epsilon_k \psi_k \quad (6)$$

$$[-i\boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + M_\Lambda + \gamma_0 \Sigma_V^\Lambda + \Sigma_S^\Lambda] \psi_\Lambda = \epsilon_\Lambda \psi_\Lambda, \quad (7)$$

where ψ_k and ψ_Λ are now the wave functions of the Kohn-Sham single particle orbits for the nucleons and the Λ , respectively. These single particle Dirac equations together with the self-energies Σ_i are solved self-consistently in the “no-sea” approximation [18]. It is important to note that rearrangement contributions Σ_R [19] in the previous equations are confined to the nucleon sector because all the density dependent couplings are polynomials in k_f (and consequently in fractional powers of the baryon density through the relation $\rho = 2 k_f^3 / (3\pi^2)$), and there is no hyperon Fermi sea. The Λ self-energies are

$$\Sigma_V^\Lambda = G_V^\Lambda(\rho) \rho, \quad \Sigma_S^\Lambda = G_S^\Lambda(\rho) \rho_S, \quad (8)$$

in terms of the nuclear baryon and scalar densities, ρ and ρ_S . Inclusion and analysis of corrections from derivative and tensor terms are postponed to a forthcoming paper.

In the following paragraphs we separately analyze the different contributions to the density dependent Λ -nuclear couplings $G_i^\Lambda(\rho)$ arising from the kaon- and two-pion-exchange induced Λ -nucleus potential, the condensate background mean fields and the pionic Λ -nucleus spin-orbit interaction.

2.2 Kaon- und two-pion induced mean field. In Ref. [13] the density dependent self-energy for a zero momentum Λ hyperon in isospin-symmetric nuclear matter has been calculated at two-loop order in the energy density. This calculation systematically includes kaon-exchange Fock terms and two-pion exchange with Σ hyperons and Pauli blocking effects in the intermediate state. This self-energy is translated into a mean field potential $U_\Lambda(k_f)$. A cutoff scale $\bar{\Lambda}$ (or equivalently, a contact term) represents short distance (high momentum) dynamics not resolved at scales characteristic of the Fermi momentum. Tuning this scale to $\bar{\Lambda} = 0.71$ GeV, remarkably close to the value 0.7 GeV used in Ref. [13], the depth of the Λ -nuclear central potential is fixed such that the p -state in ${}^{16}_\Lambda O$ is reproduced (empirically, $\epsilon_\Lambda^p = -2.5 \pm 1.0$ MeV [22])¹. We then follow the procedure outlined in the Appendix A of Ref. [15] and determine the equivalent density dependent Λ point coupling vertices $G_S^{\Lambda(\pi)}(\rho)$ and $G_V^{\Lambda(\pi)}(\rho)$. For the nucleon sector of the energy density functional we use the parameter set FKVW [15]: four parameters related to contact terms that appear in the ChPT treatment of nuclear matter, one parameter for the derivative (surface) term, and two more for the strengths of the condensate background

¹ all the calculations are carried out for ${}^{17}_\Lambda O$.

scalar and vector mean fields. In Fig. 1 (case *a*) the resulting Λ single particle energy levels of ${}^{16}_{\Lambda}O$ are plotted. At this stage the p shell spin-orbit partners are practically degenerate as expected from previous investigations [14]. The energies of the degenerate doublets are, by construction, close to their observed positions. Even the calculated energy of the s state is realistic although slightly too large in comparison with the empirical $\epsilon_{\Lambda}^s = -12.5 \pm 0.35$ MeV for ${}^{16}_{\Lambda}O$ [22].

Up to this point in-medium chiral $SU(3)$ dynamics (with K and 2π exchange) provides the necessary binding of the system but no spin-orbit force. As already shown in Ref. [15] inclusion of derivative couplings does not remove the degeneracy of the spin-orbit doublets.

2.3 Background scalar and vector mean fields. In contrast to the mean field induced by kaon and two-pion exchange, condensate background self-energies of the Λ produce a sizeable spin-orbit potential in a way analogous to what has been pointed out in Ref. [15,16] for the nucleon case. Under the assumption that only non-strange quarks are involved in interactions with the background fields, one expects a reduction of the corresponding couplings,

$$G_{S,V}^{\Lambda(0)} = \chi G_{S,V}^{(0)}, \quad (9)$$

by a factor $\chi = 2/3$ [7], where $G_V^{(0)}$ and $G_S^{(0)}$ are the vector and the scalar couplings to nucleons, arising from in-medium changes of the quark condensates, $\langle \bar{q}q \rangle$ and $\langle q^\dagger q \rangle$. The $G_V^{(0)}$ and $G_S^{(0)}$ have been determined, in good agreement with leading-order QCD sum rules estimates [20], by fitting ground state properties of finite nuclei [16].

For illustration we plot in Fig. 1 (case *b*) the Λ single particle energy levels with inclusion of these scalar and vector mean fields using $\chi = 2/3$. Now the p shell spin-orbit partners are no longer degenerate and a spin-orbit splitting of about ~ 2 MeV results. The choice $\chi = 2/3$ is, of course, a simplistic estimate. A detailed QCD sum rule analysis suggests a reduction to $\chi \sim 0.4 - 0.5$ [20,21], and to even smaller values if corrections from in-medium condensates of higher dimensions are taken into account. We shall therefore be guided by such reduced values of χ . Nonetheless, the Λ -nuclear spin-orbit force is evidently still far too strong at this level, just as in the phenomenological relativistic "sigma-omega" mean field models.

2.4 Λ -nuclear spin-orbit interaction from chiral $SU(3)$ two-pion exchange. In Ref. [13] the Λ -nucleus spin-orbit interaction generated by the in-medium two-pion exchange ΛN interaction, has been evaluated as follows. In the spin-dependent part of the self-energy of a Λ hyperon scattering in slightly inhomogeneous nuclear matter from initial momentum $\vec{p} - \vec{q}/2$ to final momentum $\vec{p} + \vec{q}/2$, one identifies a spin-orbit term, $\Sigma_{is}^{\Lambda}(k_f) = \frac{i}{2} U_{is}^{\Lambda}(k_f) \vec{\sigma} \cdot (\vec{q} \times \vec{p})$. It de-

depends only on known $SU(3)$ axial vector coupling constants and on the mass difference between Λ and Σ . The relevant momentum space loop integral is finite and hence model independent in the sense that no regularizing cutoff is required. The result, $U_{is}^\Lambda(k_f^{(0)}) \simeq -15 \text{ MeV fm}^2$ at $k_f^{(0)} \simeq 1.36 \text{ fm}^{-1}$, has a sign *opposite* to the standard nuclear spin-orbit interaction². Evidently, this term tends to largely cancel the spin-orbit potential generated by the scalar-vector background mean field.

It is important to note that such a “wrong-sign” spin-orbit interaction (generated by the second order tensor force from iterated pion exchange) exists also for nucleons in ordinary nuclei [26]. However, this effect is compensated to a large extent by the three-body spin-orbit force involving virtual $\Delta(1232)$ isobar excitation [27], so that the spin-orbit interaction from the strong scalar-vector mean fields prevails³. For a Λ hyperon, the analogous three-body effects do not exist and the compensation is now between spin-orbit terms from the (weaker) scalar-vector mean field and the in-medium second order tensor force from iterated pion exchange with intermediate Σ . The small $\Sigma\Lambda$ mass splitting, $M_\Sigma - M_\Lambda = 77.5 \text{ MeV}$, plays a prominent role in this mechanism.

In order to estimate the impact of this genuine “wrong-sign” Λ -nuclear spin-orbit term, we introduce

$$\Delta\mathcal{H}_{is}^\Lambda = -i \frac{U_{is}^\Lambda(k_f^{(0)})}{2r} \frac{df(r)}{dr} \vec{\sigma} \cdot (\vec{r} \times \vec{\nabla}) , \quad (10)$$

with the normalized nuclear density profile $f(r) = \rho(r)/\rho(r=0)$. We then evaluate the corrections to the Λ single particle energies ϵ_Λ in first order perturbation theory:

$$\epsilon'_\Lambda = \epsilon_\Lambda + \langle \phi | \Delta\mathcal{H}_{is}^\Lambda | \phi \rangle , \quad (11)$$

where $|\phi\rangle$ denotes the self-consistent solution of the system of Dirac single-baryon equations (6) and (7). In Fig. 1 (case *c*) one observes that the resulting p shell single particle energy levels, corrected according to Eq.(11), are then close to being degenerate. The spin-orbit splitting is now strongly reduced but still too large in comparison with empirical estimates. This is considered a consequence of the possibly too large quark model factor $\chi = 2/3$. Reducing this factor to the range of values compatible with QCD sum rules, the final

² Recall that nuclear Skyrme phenomenology gives $U_{is}^N(k_f^{(0)}) = 3W_0\rho_0/2 \simeq 30 \text{ MeV fm}^2$ for the nucleonic spin-orbit coupling strength [24].

³ The intimate connection between large scalar and vector mean fields of opposite sign in nuclear matter and the short-range spin-orbit term of realistic NN-potentials has recently been demonstrated in Ref. [25]

step towards the empirical, almost vanishing spin-orbit splitting for the Λ can indeed be accomplished.

In Fig. 2 we plot the Λ spin-orbit splitting $\delta_\Lambda = \epsilon_\Lambda(1p_{1/2}^{-1}) - \epsilon_\Lambda(1p_{3/2}^{-1})$ in $^{16}_\Lambda O$ as function of the ratio χ between the scalar-vector background mean fields for the Λ hyperon and for the nucleon. The filled circles show the spin-orbit splitting produced by the scalar-vector background fields alone. Even for unnaturally small values of χ , the splitting remains systematically too large in comparison with the empirical bounds. Introducing the model-independent spin-orbit contribution from second order pion exchange as in Eq.(11), the previous line is shifted downward (filled triangles) by about 1.3 MeV. With χ given in the range 0.4 - 0.5 according to the QCD sum rule analysis [20,21], the small or even vanishing spin-orbit splitting is now reproduced in agreement with empirical estimates [4,23].

3. Concluding remarks. The compensating mechanism for the spin-orbit interaction of a Λ in nuclear matter, proposed in Ref. [13], appears to be successful in explaining the very small spin-orbit splitting in finite Λ hypernuclei. We emphasize again that this mechanism, driven by the second order pion exchange tensor force between Λ and nucleon with intermediate Σ , is model-independent in that it relies only on SU(3) chiral dynamics with empirically well known constants. This well controlled intermediate-range effect (independent of any regularization procedure) counteracts short-distance spin-orbit forces. The corresponding effect in ordinary nuclei is neutralized by three-body spin-orbit terms (induced by two-pion exchange with virtual Δ -isobar excitation) which are absent in hypernuclei. Systematic applications of the present framework to larger classes of hypernuclei are under way.

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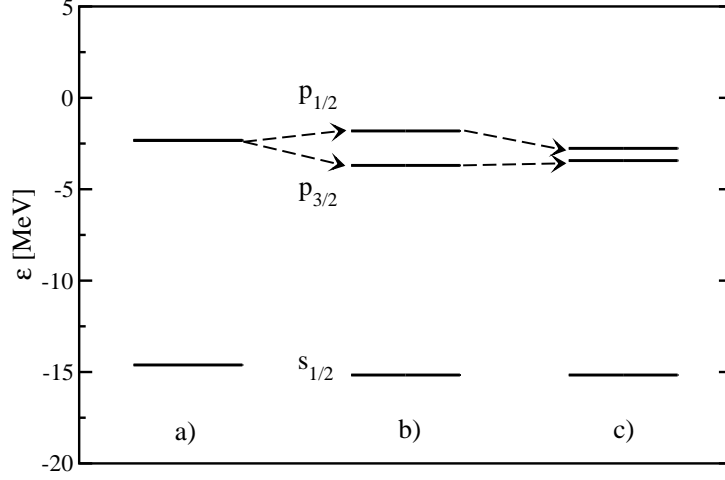


Fig. 1. Λ single particle energy levels in $^{16}_{\Lambda}O$. Case *a*: using the single particle potential based on the density dependent coupling strengths including chiral K and 2π exchange as determined in Ref. [13] (see Sect. II). Case *b*: adding the spin-orbit effect from in-medium quark condensates, with reduction factor $\chi = 2/3$ according to a simple quark model [7]. Case *c*: additional compensating effect of the chiral SU(3) spin-orbit potential from the second order ΛN tensor force with intermediate Σ (see Eq.(11)).

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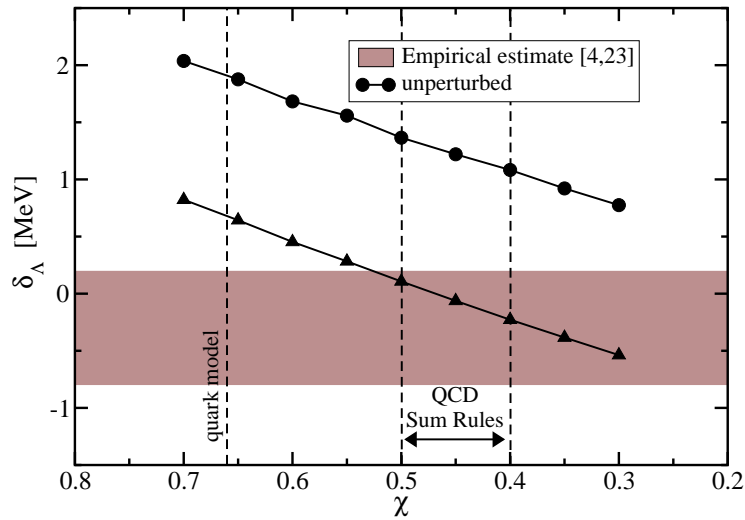


Fig. 2. Evolution of the spin-orbit splitting $\delta_\Lambda = \epsilon_\Lambda(1p_{1/2}^{-1}) - \epsilon_\Lambda(1p_{3/2}^{-1})$ for ${}^{16}_\Lambda O$ as function of the ratio χ between background scalar-vector self-energies of Λ vs. nucleon. The dashed line at $\chi = 2/3$ marks the simple quark model value. Also indicated is the χ interval suggested by a QCD sum rule analysis [20,21]. Calculations with (without) the chiral SU(3) spin-orbit correction (11) are denoted by triangles (circles). The shaded area represents an estimate of δ_Λ ($-0.8 \text{ MeV} \leq \delta_\Lambda \leq 0.2 \text{ MeV}$) based on the measured excitation energy difference $\Delta E(0^+ - 2^+)$ in ${}^{16}_\Lambda O$ [4,23]. This estimate is consistent with recent results from hypernuclear γ ray spectroscopy [5].