Abstract

Using SU(3) chiral perturbation theory we calculate the density-dependent complex-valued spin-orbit coupling strength $U_{\Sigma\Lambda}(k_f) + i W_{\Sigma\Lambda}(k_f)$ of a Σ hyperon in the nuclear medium. The leading long-range ΣN interaction arises from iterated one-pion exchange with a Λ or a Σ hyperon in the intermediate state. We find from this unique long-range dynamics a sizeable “wrong-sign” spin-orbit coupling strength of $U_{\Sigma\Lambda}(k_f_0) \approx -20\text{ MeVfm}^2$ at normal nuclear matter density $\rho_0 = 0.16\text{ fm}^{-3}$. The strong $\Sigma N \to \Lambda N$ conversion process contributes at the same time an imaginary part of $W_{\Sigma\Lambda}(k_f_0) \approx -12\text{ MeVfm}^2$. When combined with estimates of the short-range contribution the total Σ-nuclear spin-orbit coupling becomes rather weak.

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Hypernuclear physics has a long and well-documented history [1, 2, 3]. One primary goal in this field is to determine from the experimental data the nuclear mean-field potentials relevant for the hyperon single-particle motion. For the Λ hyperon the situation is by now rather clear and the following quantitative features have emerged. The attractive nuclear mean-field potential for a Λ hyperon is about half as strong as the one for nucleons in nuclei: $U_{\Lambda} \approx -28\text{ MeV}$ [4]. With this value of the potential depth the empirical single-particle energies of a Λ bound in hypernuclei are well described over a wide range in mass number. On the other hand, the Λ-nucleus spin-orbit interaction is found to be extraordinarily weak. For example, recent precision measurements [5] of $E1$-transitions from $p$- to $s$-shell orbitals in $^{13}$ΛC give a $p_{3/2} - p_{1/2}$ spin-orbit splitting of only $(152 \pm 65)\text{ keV}$ to be compared with a value of about $6\text{ MeV}$ in ordinary $p$-shell nuclei.

In case of the Σ hyperon recent developments have lead to a revision concerning the sign and magnitude of its nuclear mean-field potential [6]. Whereas an earlier analysis of the shifts and widths of x-ray transitions in Σ$^-$ atoms came up with an attractive (real) Σ-nucleus optical potential of about $-27\text{ MeV}$ [1], there is currently good experimental and phenomenological evidence for a substantial Σ-nucleus repulsion. A reanalysis of the Σ$^-$ atom data in Ref.[7] including the then available precise measurements of W and Pb atoms and employing phenomenological density-dependent fits has lead to a Σ-nucleus potential with a strongly repulsive core (of height $\sim 95\text{ MeV}$) and a shallow attractive tail outside the nucleus. The inclusive ($\pi^-, K^+$) spectra on medium-to-heavy nuclear targets measured at KEK [8, 9] give more direct evidence for a strongly repulsive Σ-nucleus potential. In the framework of the distorted wave impulse approximation, a best fit of the measured ($\pi^-, K^+$) inclusive spectra on Si, Ni, In and Bi targets is obtained with a Σ-nucleus repulsion of about $90\text{ MeV}$. However, the detailed description of the Σ$^-$ production mechanism plays an important role for the extracted value of the Σ-nucleus repulsion. Within a semiclassical distorted wave model [10], which avoids the factorization approximation by an averaged differential cross section, the KEK data can also be well reproduced with a complex Σ-nucleus potential of strength $(30 - 20i)\text{ MeV}$. Concerning the Σ-nucleus spin-orbit coupling there exist so far no experimental hints for it. Most theoretical models [11, 12] predict the Σ-nucleus spin-orbit coupling to be strong (i.e. comparable to
the one of nucleons). The basic argument for a strong spin-orbit coupling is provided by the large and positive value of the tensor-to-vector coupling ratio of the \( \omega \) meson to the \( \Sigma \) hyperon assuming vector meson dominance and the non-relativistic quark model with SU(6) spin-flavor symmetry. The G-matrix calculations by the Kyoto-Niigata group [13] using the hyperon-nucleon interaction as derived from their SU(6) quark model predict a \( \Sigma \)-nucleus spin-orbit coupling which is about half as strong as the one of nucleons. However, due to the presence of the strong \( \Sigma N \rightarrow \Lambda N \) conversion process in the nuclear medium one expects the \( \Sigma \)-nucleus spin-orbit coupling strength to have also an imaginary part. This possibility has generally been ignored in quark and one-boson exchange models.

Recently, we have applied chiral effective field theory to calculate the hyperon mean-fields in nuclear matter [14]. In this approach the small \( \Lambda \)-nuclear spin-orbit interaction finds a novel explanation in terms of an almost complete cancellation between short-range contributions (estimated from the known nucleonic spin-orbit coupling strength) and long-range terms generated by iterated one-pion exchange with intermediate \( \Sigma \) hyperons. The exceptionally small \( \Sigma \Lambda \) mass splitting of \( M_{\Sigma} - M_{\Lambda} = 77.5 \) MeV influences hereby prominently the effect coming from the second order \( 1\pi \)-exchange tensor interaction. Furthermore, it has been shown in Ref.[15] that the proposed cancellation mechanism does not get disturbed by the inclusion of analogous two-pion exchange processes involving decuplet baryons (\( \Delta (1232) \) and \( \Sigma^* (1385) \)) in the intermediate state with considerably larger mass splittings. The density-dependent complex \( \Sigma \)-nuclear mean-field \( U_{\Sigma}(k_f) + i W_{\Sigma}(k_f) \) has also been calculated in the same framework in Ref.[16]. It has been found that genuine long-range\(^1\) contributions from iterated one-pion exchange with intermediate \( \Lambda \) and \( \Sigma \) hyperons sum up to a moderately repulsive (real) single-particle potential of \( U_{\Sigma}(k_{f0}) \approx 59 \) MeV at normal nuclear matter density \( \rho_0 = 0.16 \text{fm}^{-3} \). The \( \Sigma N \rightarrow \Lambda N \) conversion process induced by one-pion exchange generates at the same time an imaginary single-particle potential of \( W_{\Sigma}(k_{f0}) \approx -21.5 \) MeV. This value is in fair agreement with empirical determinations [7] and quark model predictions [17]. The purpose of the present short paper is to calculate in the same chiral effective field theory framework the density-dependent complex-valued \( \Sigma \)-nuclear spin-orbit coupling strength. As for the \( \Lambda \) hyperon [14] we do find a sizeable “wrong-sign” spin-orbit coupling from the second-order one-pion exchange tensor interaction. When combined with estimates of the short-range contribution (employing QCD sum rule predictions) the total \( \Sigma \)-nuclear spin-orbit coupling becomes rather weak.

Let us begin with some basic considerations. The pertinent quantity to extract the \( \Sigma \)-nuclear spin-orbit coupling is the spin-dependent part of the self-energy of a \( \Sigma \) hyperon interacting with weakly inhomogeneous isospin-symmetric (spin-saturated) nuclear matter. Let the \( \Sigma \) hyperon scatter from initial momentum \( \vec{p} - \vec{q}/2 \) to final momentum \( \vec{p} + \vec{q}/2 \). The spin-orbit part of the self-energy is then:

\[
\Sigma_{\text{spin}} = \frac{i}{2} \vec{\sigma} \cdot (\vec{q} \times \vec{p}) \left[ U_{\Sigma ls}(k_f) + i W_{\Sigma ls}(k_f) \right],
\]

where the density-dependent spin-orbit coupling strength \( U_{\Sigma ls}(k_f) + i W_{\Sigma ls}(k_f) \) is taken in the limit of homogeneous nuclear matter (characterized by its Fermi momentum \( k_f \)) and zero external \( \Sigma \)-momenta: \( \vec{p} = \vec{q} = 0 \). The more familiar spin-orbit Hamiltonian follows from Eq.(1) by multiplication with a density form factor and Fourier transformation \( \int d^3q \exp(i\vec{q} \cdot \vec{r}) \). For orientation, consider first the \( \omega \) meson exchange between the \( \Sigma \) hyperon and the nucleons. The non-relativistic expansion of the vector (and tensor) coupling vertex between Dirac spinors of

\(^1\)Genuine long-range means that (unique) part of the pion-loop which depends exclusively on small scales \((k_f, m_\pi, \Delta)\), but not any high-momentum cutoff. In case of the \( \Sigma \)-nuclear mean field \( U_{\Sigma}(k_f) \) it seems that the net short-range contribution is small [16]. For the \( \Lambda \) single-particle potential \( U_{\Lambda}(k_f) \) an attractive short-range contribution [14] is however necessary in order to reproduce the empirical potential depth of \(-28 \) MeV. A deeper understanding of this feature is presently missing.
the Σ hyperon gives rise to a spin-orbit term proportional to $i \vec{\sigma} \cdot (\vec{q} \times \vec{p})/4M_\Sigma^2$. Next one takes the limit of homogeneous nuclear matter (i.e. $\vec{q} = 0$), performs the remaining integral over the nuclear Fermi sphere and arrives at the familiar result:

$$U_{\Sigma ls}(k_f) = \frac{g_{\omega \Sigma} (1 + 2\kappa_{\omega \Sigma}) g_{\omega N}}{2M_\Sigma^2 m_\omega^2} \rho,$$

linear in density $\rho = 2k^3/3\pi^2$. Here, $\kappa_{\omega \Sigma}$ denotes the tensor-to-vector coupling ratio of the $\omega$ meson to the $\Sigma$ hyperon.

The crucial observation is now that the (left) iterated one-pion exchange diagram in Fig. 1 generates also a (sizeable) spin-orbit coupling term. The prefactor $\frac{i}{2} \vec{\sigma} \times \vec{q}$ is immediately identified by rewriting the product of $\pi \Sigma B$-interaction vertices $\vec{\sigma} \cdot (\vec{l} - \vec{q}/2) \vec{\sigma} \cdot (\vec{l} + \vec{q}/2) = \frac{i}{2} (\vec{\sigma} \times \vec{q}) \cdot (-2\vec{l}) + \ldots$ at the open baryon line. For all remaining parts of the diagram one can then take the limit of homogeneous nuclear matter (i.e. $\vec{q} = 0$). The other essential factor $\vec{p}$ comes from the energy denominator $-\Delta^2 + \vec{l} \cdot (\vec{l} - \vec{p}_1 + \vec{p})$. The $\Sigma \Lambda$ mass splitting is rewritten here in terms of the small scale parameter $\Delta = \sqrt{M_B(M_\Sigma - M_\Lambda)} \approx 285$ MeV with $M_B = (2M_N + M_\Lambda + M_\Sigma)/4 \approx 1047$ MeV a mean baryon mass. It serves the purpose to average out small differences in the kinetic energies of the various baryons involved. Keeping only the term linear in the external momentum $\vec{p}$ one finds from the left diagram in Fig. 1 with a $\Lambda$ hyperon in the intermediate state the following contribution to the $\Sigma$-nuclear spin-orbit coupling strength:

$$U_{\Sigma ls}(k_f)^{(2\pi A)} + i W_{\Sigma ls}(k_f)^{(2\pi A)} = -\frac{2D^2 g_A^2}{9f_\pi^4} \int \frac{d^3p_1 d^3l}{|p_1| < k_f} \frac{M_B \vec{l}^4}{(2\pi)^6 (m_{\pi}^2 + \vec{l}^2)^2 [-\Delta^2 - i0 + \vec{l}^2 - \vec{l} \cdot \vec{p}_1]^2}$$

$$= \frac{2}{3} \frac{\partial}{\partial \Delta^2} \left[ U_{\Sigma}(k_f)^{(2\pi A)} + i W_{\Sigma}(k_f)^{(2\pi A)} \right].$$

Here, $D = 0.84$ and $F = 0.46$ [14] denote the SU(3) axial vector coupling constants together with $g_A = D + F = 1.3$ the nucleon axial vector coupling constant. $f_\pi = 92.4$ MeV is the pion decay constant and $m_{\pi} = 138$ MeV the average pion mass. Note that the loop integral in Eq.(3) is convergent as its stands. Most useful is actually the representation of the spin-orbit coupling strength as a derivative of the $\Sigma$-nuclear potential $U_{\Sigma}(k_f) + i W_{\Sigma}(k_f)$ with respect to the (mass splitting) parameter $\Delta^2$. Using the analytical expressions in Ref.[16] to evaluate this derivative
we find for the real and imaginary part:

\[
U_{\Sigma ls}(k_f)_{(2\pi \Lambda)} = \frac{D^2 g_A^2 M_B m_\Sigma^2}{72 \pi^3 f_\pi^4} \left\{ (4 + 2\delta) \arctan \frac{\sqrt{u}}{1 + \delta} - \frac{3u + (1 + \delta)(4 + 2\delta)}{u + (1 + \delta)^2} \sqrt{u} \right\}, \tag{4}
\]

\[
W_{\Sigma ls}(k_f)_{(2\pi \Lambda)} = \frac{D^2 g_A^2 M_B m_\Sigma^2}{72 \pi^3 f_\pi^4} \left\{ -\frac{u + (1 + \delta)(2 + \delta)}{u + (1 + \delta)^2} \sqrt{u(4\delta + u)} + (4 + 2\delta) \ln \frac{u + 2 + 2\delta + \sqrt{u(4\delta + u)}}{2[u + (1 + \delta)^2]^{1/2}} \right\}, \tag{5}
\]

with the abbreviations \( u = k_f^2/m_\pi^2 \) and \( \delta = \Delta^2/m_\pi^2 \). The right diagram in Fig.1 with two medium insertions represents the Pauli blocking correction. In comparison to the expression in Eq.(3) the sign is reverse and the momentum transfer \( \vec{t} \) gets replaced by \( \vec{t} = \vec{p}_1 - \vec{p}_2 \) with \( \vec{p}_2 \) to be integrated over a Fermi sphere of radius \( k_f \), i.e. \( |\vec{p}_2| < k_f \). In case of the real part one is left with a double-integral of the form:

\[
U_{\Sigma ls}(k_f)_{(2\pi \Lambda)_{\text{Pauli}}} = \frac{D^2 g_A^2 M_B m_\Sigma^2}{36 \pi^4 f_\pi^4} \int_0^u dx \int_0^u dy \frac{1}{(2\delta + 1 + x - y)^2} \left\{ \frac{(2\delta + x - y)^2 \sqrt{xy}}{2(\delta - y)^2 - 2xy} + \frac{2\sqrt{xy}}{(1 + x + y)^2 - 4xy} + \frac{2\delta + x - y}{2\delta + 1 + x - y} \ln \frac{|\delta - y - \sqrt{xy}(1 + x + y - 2\sqrt{xy})|}{|\delta - y + \sqrt{xy}(1 + x + y + 2\sqrt{xy})|} \right\}, \tag{6}
\]

where the first term in brackets has to be treated as a principal value integral. In practice this is done by solving the \( \int_0^u dx \)-integral analytically and converting the occurring logarithms into logarithms of absolute values. The Pauli blocking correction to the imaginary part \( W_{\Sigma ls}(k_f) \) can even be written in closed analytical form:

\[
W_{\Sigma ls}(k_f)_{(2\pi \Lambda)_{\text{Pauli}}} = \frac{D^2 g_A^2 M_B m_\Sigma^2}{72 \pi^3 f_\pi^4} \theta(\sqrt{2k_f} - \Delta) \left\{ \frac{u}{2} - \delta - 1 + \frac{1}{1 + 2\delta} + \frac{u\delta}{u + \delta^2} + \frac{u(1 - \delta)}{2u + 2(1 + \delta)^2} + \frac{u + (1 + \delta)(2 + \delta)}{2u + 2(1 + \delta)^2} \sqrt{u(4\delta + u)} + 2 \ln(2 + 4\delta) + \delta \ln(2 + 2\delta u^{-1}) - (2 + \delta) \ln \left[u + 2 + 2\delta + \sqrt{u(4\delta + u)}\right] \right\}. \tag{7}
\]

Interestingly, there is a threshold condition \( k_f > \Delta/\sqrt{2} \) for Pauli blocking to become active in the imaginary part. The threshold opens at about one half of nuclear matter saturation density \( \rho_{\text{th}} = 0.072 \text{fm}^{-3} = 0.45 \rho_0 \).

The additional contributions from the iterated one-pion exchange diagrams with a \( \Sigma \) hyperon in the intermediate state are obtained by substituting axial vector coupling constants, \( D^2 \rightarrow 6F^2 \), and dropping the \( \Sigma \Lambda \) mass splitting, \( \delta \rightarrow 0 \). The explicit expressions for these contributions to the complex \( \Sigma \)-nuclear spin-orbit coupling strength read:

\[
U_{\Sigma ls}(k_f)_{(2\pi \Sigma)} = \frac{F^2 g_A^2 M_B m_\Sigma^2}{12 \pi^3 f_\pi^4} \left\{ 4 \arctan \left[ \frac{\sqrt{u}}{1 + u} \sqrt{u} \right] - \frac{4 + 3u}{1 + u} \right\}, \tag{8}
\]

\[
W_{\Sigma ls}(k_f)_{(2\pi \Sigma)} = -W_{\Sigma ls}(k_f)_{(2\pi \Lambda)_{\text{Pauli}}} = \frac{F^2 g_A^2 M_B m_\Sigma^2}{12 \pi^3 f_\pi^4} \left\{ 2 \ln(1 + u) - \frac{2u + u^2}{1 + u} \right\}, \tag{9}
\]
\[ U_{\Sigma ls}(k_f)^{2n\Sigma}_{\text{Pauli}} = \frac{F^2 g^2 M_B m_n^2}{12\pi^4 F^4} \left\{ 6\sqrt{u} \arctan(2\sqrt{u}) - 2u - \frac{2\sqrt{u}}{\sqrt{1+u}} \ln(\sqrt{u} + \sqrt{1+u}) \right\} \\
- \frac{3}{2} \ln(1+4u) + \int_0^u \left[ \frac{1+2u-2x}{(1-u-x)^2} \ln \left( \frac{\sqrt{u} - \sqrt{x}}{(1+u+x-2\sqrt{ux})} \right) \right] dx \]

where now almost all integrals could be solved for the Pauli blocking correction.

Summing up all calculated two-loop terms written in Eqs.(4-10) we show in Fig. 2 the resulting complex Σ-nuclear spin-orbit coupling strength \( U_{\Sigma ls}(k_f) + i W_{\Sigma ls}(k_f) \) as a function of the nucleon density in the region \( 0 \leq \rho \leq 0.2 \text{fm}^{-3} \) (corresponding to Fermi momenta \( k_f \leq 283 \text{MeV} \)). It is expected that higher-loop contributions related to pion-absorption on two nucleons, in-medium nucleon and pion self-energy corrections etc. are small in this low-density region. The upper curve for the imaginary part \( W_{\Sigma ls}(k_f) \) clearly displays the onset of the Pauli blocking effect at the threshold density \( \rho_{th} = 0.072 \text{fm}^{-3} \). It may come as a surprise that Pauli blocking increases the magnitude of the negative imaginary part. But going back to the original expression Eq.(3) one sees that the squared energy denominator introduces as a weight function for imaginary part the derivative of a delta-function. Therefore the usual argument of phase space reduction by Pauli blocking becomes insufficient even for a qualitative estimate. At normal nuclear matter density \( \rho_0 = 0.16 \text{fm}^{-3} \) (corresponding to a Fermi momentum of \( k_{f0} = 263 \text{MeV} \)) one finds for the total imaginary part \( W_{\Sigma ls}(k_{f0}) = (-6.83 - 4.89) \text{MeVfm}^2 = -11.7 \text{MeVfm}^2 \), where the second entry stems from Pauli blocking. The physics behind this imaginary spin-orbit coupling strength is, of course, the \( \Sigma N \rightarrow \Lambda N \) conversion process induced by \( 1\pi \)-exchange. One can also see from Fig. 2 that the cusp effect in the imaginary part \( W_{\Sigma ls}(k_f) \) causes some non-smooth behavior of the real part \( U_{\Sigma ls}(k_f) \). The almost linear decrease with density gets interrupted at the threshold density \( \rho_{th} = 0.072 \text{fm}^{-3} \). At saturation density one finds a “wrong-sign” Σ-nuclear spin-orbit coupling strength of \( U_{\Sigma ls}(k_{f0}) = [(−1.83 − 2.32) + (−18.21 + 2.43)] \text{MeVfm}^2 = −19.9 \text{MeVfm}^2 \), where the individual entries correspond to respective terms written in Eqs.(4,6,8,10), in that order. It is somewhat larger than the “wrong-sign” spin-orbit coupling of a Λ hyperon, \( U_{\Lambda ls}(k_{f0}) = −15 \text{MeVfm}^2 \). This is our major result: The second order \( 1\pi \)-exchange tensor interaction generates sizeable “wrong-sign” spin-orbit couplings for the Λ and the Σ hyperon together. The negative sign in case of the Σ hyperon is however less obvious, because the relevant loop integrals are derivatives of six-dimensional principal value integrals (see Eq.(3)). As an aside we note that in the chiral limit \( (m_\pi = 0) \) the Σ-nuclear spin-orbit coupling strength changes to \( U_{\Sigma ls}(k_{f0}) + i W_{\Sigma ls}(k_{f0}) = (−25.0 − 13.0i) \text{MeVfm}^2 \), with the real part coming now entirely from the Pauli blocking corrections.

It is expected that the additional \( 2\pi \)-exchange effects of Ref.[15] including decuplet baryons in the intermediate state do not change the present results in a significant way. First, the additional mass splittings in the energy denominators are so high that no new contribution to the imaginary part \( W_{\Sigma ls}(k_f) \) is generated for \( \rho \leq \rho_0 \). Secondly, the approximate cancellation between the contributions from \( \Delta(1232) \) and \( \Sigma^*(1385) \) intermediate states works for Λ and Σ hyperons together, since it is based on different signs of spin-sums [15].

The short-range part of the Σ-nuclear spin-orbit interaction results from a variety of processes, one of them being the \( \omega \)-exchange piece presented in Eq.(2). Following Ref.[14], we relate the short-distance spin-orbit coupling of the Σ hyperon to the one of the nucleon as follows:

\[ U_{\Sigma ls}(k_f)^{\text{sh}} = C_{ls} \frac{M_N^2}{M_\Sigma^2} U_{N ls}(k_f)^{\text{sh}}. \]

The factor \( (M_N/M_\Sigma)^2 = 0.62 \) results from the replacement of the nucleon by a Σ hyperon in
these relativistic spin-orbit terms. The coefficient $C_{ls}$ parameterizes the ratio of the relevant coupling constants. The expectation from the naive quark model would be $C_{ls} = 2/3$. On the other hand, QCD sum rule calculations of $\Sigma$ hyperons in nuclear matter [18] indicate that the Lorentz scalar and vector mean fields of a $\Sigma$ hyperon are similar to the corresponding ones of a nucleon, i.e. $C_{ls} \simeq 1$. In case of the Lorentz scalar mean field, the QCD sum rule calculations are subject to uncertainties due to poorly known contributions from four-quark condensates. Ref.[18] concludes that due to a significant SU(3) symmetry breaking in nuclear matter the short-range spin-orbit term of a $\Sigma$ hyperon may be comparable to the one of a nucleon. For the further discussion we take for the short-range nucleonic spin-orbit coupling strength $U_{\Sigma ls}(k_f) = 3\rho W_0/2 = 30\text{MeVfm}^2\rho/\rho_0$ with $W_0 = 124\text{MeVfm}^5$ the spin-orbit parameter in the Skyrme phenomenology [19]. Employing $C_{ls} \simeq 1$, as indicated by the sum rule calculations, one estimates the short-range $\Sigma$-nuclear spin-orbit coupling strength to $U_{\Sigma ls}(k_{f0}) \simeq 18.6\text{MeVfm}^2$. This would lead to an almost complete cancellation of the long-range component generated by iterated one-pion exchange, resulting in a rather weak $\Sigma$-nuclear spin-orbit coupling (admittedly with large uncertainties). Finally, we note that the long-range and short-range pieces are distinguished by markedly different dependences on the pion mass $m_\pi$ (or light quark mass $m_q \sim m_\pi^2$) and the density $\rho = 2k_f^3/3\pi^2$. Therefore, there seems to be no double counting problem when adding long-range and short-range components.

In summary, we have calculated in this work the $\Sigma$-nuclear spin-orbit coupling generated by iterated one-pion exchange with a $\Lambda$ or a $\Sigma$ hyperon in the intermediate state. We find from this unique long-range dynamics a sizeable “wrong-sign” spin-orbit coupling strength of $U_{\Sigma ls}(k_{f0}) \simeq -20\text{MeVfm}^2$. When combined with estimates of the short-range component a weak $\Sigma$-nuclear spin-orbit coupling will result in total. Unfortunately, the prospects for an experimental check of this feature are poor. The recently established repulsive nature of the $\Sigma$-nucleus optical potential [6] precludes a rich spectroscopy of heavy $\Sigma$-hypernuclei which could
reveal spin-orbit splittings.

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References