The QCD phase diagram: PNJL model with diquarks

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Outline

- QCD phase structure
  - Spontaneous chiral symmetry breaking
  - Confinement
- Joining the NJL model and the Polyakov loop model
- Diquark degrees of freedom
- Current quark mass dependence of the tri-critical point

1. The PNJL-model
   - Combining NJL model to the Polyakov loop model
   - The PNJL model with diquarks

2. Results
   - Parameter fixing
   - Quark mass dependence of the critical point
   - Beyond mean field theory
Combining NJL model to the Polyakov loop model

Part 1: The Nambu and Jona-Lasinio model

- Local 4 quark point interaction
- Gluons are integrated out
- 3 Parameters: Cutoff \( \Lambda \), quark-quark coupling constant \( G \) and current quark mass \( m_0 \)

Spontaneous broken chiral symmetry

- Dynamic generation of quark masses
- Spontaneous chiral symmetry breaking
- Parameters chosen such that \( m_\pi \), \( f_\pi \) and \( \langle \bar{q}q \rangle \) are reproduced
The Polyakov loop:

$$L(\vec{x}) = \mathcal{P} \exp \left\{ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right\}$$

Order parameter in pure gluonic systems

Mean field $\Phi = \frac{1}{N_c} \text{tr} L$

Note: Both $\Phi$ and $L$ often referred to as Polyakov loop

Confinement encoded

- Effective potential $U(\Phi, \Phi^*, T)$

Confinement mechanism in PNJL models

- Replace $\partial_\mu \longrightarrow D_\mu = \partial_\mu - i g A_\mu^a \frac{\lambda_a}{2}$ in the Nambu-Gorkov propagator
**Choice of the form of the effective loop potential**

\[
\frac{U(\Phi, \Phi^*, T)}{T^4} = -\frac{1}{2} b_2(T) (\Phi \Phi^*) - b_3 \left( \Phi^3 + \Phi^{*3} \right) + b_4 (\Phi \Phi^*)^2
\]

- Constraint by Stefan-Boltzmann limit
- \(\lim_{T \to \infty} \langle \Phi \rangle = 1\)

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**Graphs**

- Boyd et al. (1996)
  - *Nucl. Phys.* **B469**, 419

- Kaczmarek et al. (2002)
  - *Phys. Lett.* **B543**, 41

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The PNJL model with diquarks for \( N_f = 2 \)

**Ansatz: Color-current interaction term**

\[
\delta \mathcal{L}_{\text{int.}} = -g \left( \bar{\psi} \gamma_\mu \lambda_a \psi \right) \left( \bar{\psi} \gamma_\mu \lambda_a \psi \right)
\]

**Effective Lagrangian**

\[
\mathcal{L}_{\text{PNJL}} = \bar{\psi} \left( i \gamma_\mu D^\mu - \hat{m}_0 \right) \psi + \frac{G}{2} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \bar{\tau} \psi)^2 \right] + \frac{G_8}{2} \left[ (\bar{\psi} \lambda_8 \psi)^2 + (\bar{\psi} i \gamma_5 \lambda_8 \bar{\tau} \psi)^2 \right] + \frac{H}{2} \left[ \left( \psi^T C \gamma_5 \tau_2 \lambda_2 \psi \right) \left( \bar{\psi} C \gamma_5 \tau_2 \lambda_2 \bar{\psi}^T \right) \right] - U(\Phi, \Phi^*, T)
\]

- Fierz transformation fixes the ratio
  \( G : G^{(8)} : H = 1 : \frac{3}{16} : \frac{3}{4} \)
- \( \mathcal{L}_{\text{PNJL}} \) includes direct and exchange terms

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The thermodynamic potential

\[ \Omega = U(\Phi, \Phi*, T) + \frac{\sigma^2}{2G} + \frac{\sigma_8^2}{2G_8} + \frac{|\Delta|^2}{2H} - \frac{T}{2} \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} \text{Tr} \log \frac{S^{-1}(\omega_n, \bar{p})}{T} \]

\[ \sigma = \langle \bar{q}q \rangle \quad \sigma_8 = \langle \bar{q}\lambda_8 q \rangle \quad \Delta = \langle q^T C i \gamma_5 \tau_2 \lambda_2 q \rangle \]

- Matsubara frequencies \( \omega_n = (2n + 1) \pi T \)
- Inverse Nambu Gorkov propagator

\[ S^{-1} = \begin{pmatrix} \bar{p} - m - \gamma_0 (\mu + iA_4) & \Delta \gamma_5 \tau_2 \lambda_2 \\ -\Delta^* \gamma_5 \tau_2 \lambda_2 & \bar{p} - m + \gamma_0 (\mu + iA_4) \end{pmatrix} \]

The mean field equations:

\[ \frac{\partial \Omega}{\partial \sigma} = \frac{\partial \Omega}{\partial \sigma_8} = \frac{\partial \Omega}{\partial \Delta} = \frac{\partial \Omega}{\partial A_4^{(3)}} = \frac{\partial \Omega}{\partial A_4^{(8)}} = 0 \]
Parameter fixing

Parameter choice

- $G = 10.08 \text{ MeV}$
- $\Lambda = 651 \text{ MeV}$
- $m_0 = 5.5 \text{ MeV}$

$\iff$

- $m_\pi = 140.5 \text{ MeV}$
- $f_\pi = 94.0 \text{ MeV}$
- $|\langle \bar{\psi}\psi \rangle |^{1/3} = 251 \text{ MeV}$

Variation of the current quark mass $m_0$

- Comparison with lattice data: Large quark masses.
- Reasonable agreement of $m_\pi^2$-dependence of $f_\pi$ and $M_N \approx 3 M$ with lattice result
Comparison NJL vs. PNJL at $\mu \neq 0$

- PNJL shows much better agreement with lattice data
  - The lattice data was scaled to the Stefan-Boltzmann limit
The phase diagram for varying quark mass $m_0$

Part 1:

$m_0 = 5.5 \text{ MeV}$
$T_{\text{deconf.}} = 270 \text{ MeV}$

$m_0 = 50.\text{ MeV}$
$T_{\text{deconf.}} = 270 \text{ MeV}$

Numerical precision in both $T$ and $\mu$ direction $\approx 3 \text{ MeV}$
The phase diagram for varying quark mass $m_0$
Part 2: Adjusted deconfinement transition temperature $T_{\text{deconf.}}$

$m_0 = 5.5\,\text{MeV}$
$T_{\text{deconf.}} = 190 \, \text{MeV}$

$m_0 = 50.\,\text{MeV}$
$T_{\text{deconf.}} = 190 \, \text{MeV}$
Quark mass dependence of the critical point
Comparison with lattice data

Discrepancy between PNJL ($N_f = 2$) and lattice ($N_f = 2 + 1$)
- in chemical potential:
  (see figure)
- in temperature:
  $T_{c\text{lat}} \approx 1.8 \ T_{c\text{PNJL}}$

Strangeness not yet included in PNJL calculations

Fodor & Katz (2002) JHEP 03, 014
Fodor & Katz (2004) JHEP 04, 050
**Problem:** PNJL model in mean field approximation is constraint to $\Phi = \Phi^* \in \mathbb{R}$

- Expand action to 2\textsuperscript{nd} order
- Integrate over the fields

$$\Rightarrow \langle \Phi \rangle, \langle \Phi^* \rangle \in \mathbb{R}, \langle \Phi \rangle \neq \langle \Phi^* \rangle \text{ at } \mu \neq 0$$  
(A. Dimitru, et al. (2005) PRD72, 065008)
Beyond mean field theory

Part 2: Comparison pressure difference and quark density at $\mu \neq 0$

- Better agreement with *steep rise* of lattice data
- Very good description below the transition
  - The lattice data was scaled to the Stefan-Boltzmann limit
Summary

- **PNJL:**
  - Chiral symmetry breaking
  - Confinement
- First calculation of PNJL model including diquarks
- Sensitive dependence on the quark mass
- Quark mass dependence of the critical point

- **Outlook**
  - Beyond mean field approximation
  - $2+1$ flavors