Phases of Nuclear Matter

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Nuclear Chiral Thermodynamics

- QCD interface with nuclear physics:
  Chiral Effective Field Theory
- Nuclear Equation of State and QCD phase diagram
- Density and temperature dependence of the Chiral (Quark) Condensate
- New constraints from Neutron Stars
NUCLEAR MATTER and QCD PHASES

**nuclei**

![Image of a nucleus with dashed lines pointing to different phases on a phase diagram.]

**Scales in nuclear matter:**

- **momentum scale:**
  - **Fermi momentum**
  \[ k_F \simeq 1.4 \text{ fm}^{-1} \sim 2m_\pi \]
- **NN distance:**
  \[ d_{NN} \simeq 1.8 \text{ fm} \sim 1.3 \text{ m}_\pi \]
- **energy per nucleon:**
  \[ E/A \simeq -16 \text{ MeV} \]
- **compression modulus:**
  \[ K = (260 \pm 30) \text{ MeV} \sim 2m_\pi \]
Nuclear Forces
- recent developments -

contemporary approach:

Chiral Effective Field Theory + Lattice QCD

Early history: M. Taketani et al. (1951)

Hierarchy of SCALES

contact terms

explicit treatment of two-pion exchange

definite core and tensor force from Lattice QCD

m_π = 0.53 GeV

S. Aoki, T. Hatsuda, N. Ishii
**CHIRAL EFFECTIVE FIELD THEORY**

- Systematic framework at interface of QCD and Nuclear Physics
- Interacting systems of **PIONS** (light / fast) and **NUCLEONS** (heavy / slow):

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, ...) \]

\[ U(x) = \exp\left[i\tau_a \pi_a(x)/f_\pi\right] \]

- Construction of Effective Lagrangian: **Symmetries**

  short distance dynamics: contact terms

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NUCLEAR INTERACTIONS from CHIRAL EFFECTIVE FIELD THEORY

Systematically organized HIERARCHY
Explicit $\Delta (1230)$ DEGREES of FREEDOM

- **Large spin-isospin polarizability** of the Nucleon

  example: polarized Compton scattering

  \[
  \beta_\Delta = \frac{g_A^2}{f_\pi^2 (M_\Delta - M_N)} \sim 5 \text{ fm}^3
  \]

  \[
  M_\Delta - M_N \simeq 2 m_\pi << 4\pi f_\pi
  \]

  (small scale)

- **Pionic Van der Waals** - type intermediate range central potential

  N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259

  \[
  V_c(r) = -\frac{9 g_A^2}{32\pi^2 f_\pi^2} \beta_\Delta e^{-2m_\pi r} \frac{P(m_\pi r)}{r^6}
  \]

  strong 3-body interaction

  J. Fujita, H. Miyazawa (1957)
  Pieper, Pandharipande, Wiringa, Carlson (2001)
Explicit $\Delta(1230)$ DEGREES of FREEDOM (contd.)

<table>
<thead>
<tr>
<th>Standard Chiral EFT</th>
<th>Including $\Delta$ as an explicit DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>$X \bar{H}$</td>
</tr>
<tr>
<td>NLO</td>
<td>$X \bar{H} + \underbrace{\text{Other Diagrams}}_{g_A}$</td>
</tr>
<tr>
<td>$N^2$LO</td>
<td>$\underbrace{\text{Other Diagrams}}<em>{h_A}$ + $\underbrace{\text{Other Diagrams}}</em>{b_3 + b_8}$</td>
</tr>
</tbody>
</table>

- Important physics of $\Delta(1230)$ promoted to NLO
- Improved convergence

Kaiser et al., Ordonez et al.
Krebs, Epelbaum, Meißner (2007)
Important pieces of the CHIRAL NUCLEON-NUCLEON INTERACTION

**ISOVECTOR TENSOR FORCE**

\[ V_T \]

\[ S_1 \rightarrow S_2 \]

**note: no \( \rho \) meson**

**CENTRAL ATTRACTION** from TWO-PION EXCHANGE

\[ \Delta(1232) \]

**Van der WAALS - like force:**

\[ V_c(r) \propto -\frac{\exp[-2m_\pi r]}{r^6} P(m_\pi r) \]

... at intermediate and long distance

**note: no \( \sigma \) boson**
CHIRAL DYNAMICS and the NUCLEAR MANY-BODY PROBLEM


- Small scales:
  \[ k_F \sim 2 m_\pi \sim M_\Delta - M_N << 4\pi f_\pi \]

- PIONS (and DELTA isobars) as explicit degrees of freedom

IN-MEDIUM CHIRAL PERTURBATION THEORY

Pion exchange processes in presence of filled Fermi sea

\[ \pi \text{ exchange processes in } \cdots \]

2nd order TENSOR force + nucleon’s SPIN-ISOSPIN polarizability

Short-distance dynamics: contact interactions (incl. resummations)

\[ N \times N \]
**IN-MEDIUM CHIRAL PERTURBATION THEORY**

- "Medium insertion" in the nucleon propagator:
  
  \[
  (\gamma_\mu p^\mu + m_N) \left[ \frac{i}{p^2 - m_N^2 + i\varepsilon} - 2\pi \delta(p^2 - m_N^2) \theta(p^0) \theta(k_F - |\vec{p}|) \right]
  \]

- **Loop expansion** of (In-Medium) Chiral Perturbation Theory

  - Systematic expansion of **ENERGY DENSITY** \( \mathcal{E}(k_F) \) in **powers** of Fermi momentum [modulo functions \( f_n(k_F/m_\pi) \)]
  
  \( \text{(works for } k_F << 4\pi f_\pi \sim 1 \text{ GeV)} \)

- **Nuclear thermodynamics**: compute **free energy density**

  - **(3-loop order)**
  

- **in-medium** nucleon propagators incl. Pauli blocking
In-medium ChPT
3-loop \((\pi, N, \Delta)\)

Input parameters:
two contact terms

basically:
analytic calculation

Output:

- Binding & saturation
  \[ E_0/A = -16 \text{ MeV} , \quad \rho_0 = 0.16 \text{ fm}^{-3} , \quad K = 290 \text{ MeV} \]

- Realistic (complex, momentum dependent) single-particle potential
  ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)

- Asymmetry energy: \[ A(k_F^0) = 34 \text{ MeV} \]

- Quasiparticle interaction and Landau parameters

S. Fritsch, N. Kaiser, W.W.

J.W. Holt, N. Kaiser, W.W.
Nucl. Phys. A 870 (2011) 1,
Nucl. Phys. A 876 (2012) 61,
NUCLEAR THERMODYNAMICS

NUCLEAR CHIRAL (PION) DYNAMICS

BINDING & SATURATION:

Van der Waals + Pauli

\[ \pi \]

N, \Delta

+ 3-body forces

\[ N \quad N \]

contact terms

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nuclear matter: equation of state

\[ P \text{[MeV/fm}^3\text{]} \]

\[ \rho \text{[fm}^{-3}\text{]} \]

3-loop in-medium ChEFT

\[ T = 25 \text{ MeV} \quad 20 \]

\[ T = 15 \]

\[ T = 10 \]

\[ T = 5 \]

\[ T = 0 \]

Liquid - Gas Transition at
Critical Temperature \( T_c = 15 \text{ MeV} \)
(empirical: \( T_c = 16 - 18 \text{ MeV} \))
**PHASE DIAGRAM of NUCLEAR MATTER**

- **In-medium**
  **chiral effective field theory**
  (3-loop calculation of free energy density)

  S. Fritsch, N. Kaiser, W.W.

  S. Fiorilla, N. Kaiser, W.W.

- Pion-nucleon dynamics including delta isobars
- Short-distance NN contact terms
- Three-body forces

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**Diagram:**
- **Critical point**
- **Gas**
- **Liquid**
- **Symmetric nuclear matter (N = Z)**

**Axes:**
- Temperature \( T \) [MeV]
- Baryon chemical potential \( \mu_B \) [MeV]

**Graph:**
- **Gas**
- **Liquid**
- **Phase transition**

**Legend:**
- **Gas**
- **Liquid**
- **Critical point**
- **Symmetric nuclear matter**

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**Equations:**

\[
N = Z
\]

**Notes:**
- Phase diagram of nuclear matter
- In-medium calculations
- Contributions from various researchers

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PHASE DIAGRAM of NUCLEAR MATTER

Trajectory of CRITICAL POINT for asymmetric matter
as function of proton fraction Z/A

... determined almost completely by isospin dependent (one- and two-) pion exchange dynamics

S. Fiorilla, N. Kaiser, W.W.
In-medium chiral effective field theory (3-loop) with resummation of short distance contact terms (large \( n n \) scattering length, \( a_s = 19 \, \text{fm} \))

Neutron matter behaves almost like a unitary Fermi gas

Bertsch parameter

\[
\xi = \frac{\overline{E}}{E_{\text{Fermi gas}}} \simeq 0.5
\]

perfect agreement with sophisticated many-body calculations
(e.g. VCS (Urbana) or QMC methods (P. Armani et al., arXiv:1110.0993))


Akmal, Pandharipande, Ravenhall

**Quasiparticle interaction** based on accurate NNLO chiral nucleon-nucleon interaction including three-body forces:

\[
\delta E = \sum_{\vec{p} \; \text{st}} \epsilon_{\vec{p}} \delta n_{\vec{p} \; \text{st}} + \frac{1}{2} \sum_{\vec{p}_1 s_1 t_1 \; \vec{p}_2 s_2 t_2} \mathcal{F}(\vec{p}_1 s_1 t_1; \vec{p}_2 s_2 t_2) \delta n_{\vec{p}_1 s_1 t_1} \delta n_{\vec{p}_2 s_2 t_2} + \cdots,
\]

\[
\mathcal{F}(\vec{p}_1, \vec{p}_2) = f(\vec{p}_1, \vec{p}_2) + g(\vec{p}_1, \vec{p}_2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + h(\vec{p}_1, \vec{p}_2) S_{12}(\hat{q}) + \ldots
\]

**Chiral Fermi Liquid Approach to Neutron Matter**

- **V(low-k) + 3N**
- **chiral NN + 3N**

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V. We end with a summary and conclusions.
CHIRAL CONDENSATE: 
DENSITY and TEMPERATURE DEPENDENCE

- Free energy density
  \[ \mathcal{F}(m_q; \rho, T) \]

- In-medium Chiral Effective Field Theory
  (NLO 3-loop)
  constrained by realistic nuclear equation of state

- No indication of first order chiral phase transition for
  \[ \rho \lesssim 2 \rho_0, \quad T \lesssim 100 \text{ MeV} \]

\[ \langle \Psi \mid \bar{q} q \mid \Psi \rangle_{\rho, T} = \frac{\partial \mathcal{F}(m_q; \rho, T)}{\partial m_q} \]

\[ \frac{\langle \bar{q} q \rangle_{\rho, T}}{\langle \bar{q} q \rangle_0} \]


\[ \langle \bar{q} q \rangle_0 \]

symmetric nuclear matter
\[ N = Z \]

\[ T = 0 \]

\[ T = 100 \text{ MeV} \]

\[ \rho \text{ [fm}^{-3}\text{]} \]

\[ \rho \text{ [fm}^{-3}\text{]} \]

\[ \rho \text{ [fm}^{-3}\text{]} \]
CHIRAL CONDENSATE:
Dependence on TEMPERATURE and BARYON CHEMICAL POTENTIAL

- Liquid-gas phase transition leaves its signature also in chiral condensate
- but: no tendency toward chiral first order transition in the range $\mu_B \lesssim 1$ GeV
**LIQUID - GAS TRANSITION**  
**and**  
**CHEMICAL FREEZE-OUT**


**Chiral nucleon - meson model**

\[
\mathcal{L} = \bar{\psi}_a i \gamma^\nu (\partial_\nu - i g \omega_\nu - i \mu \delta_{0\nu}) \psi_a \\
+ \sqrt{2} h [\bar{\psi}_a (\frac{1+\gamma_5}{2}) \phi_{ab} \psi_b + \bar{\psi}_a (\frac{1-\gamma_5}{2}) (\phi^\dagger)_{ab} \psi_b] \\
+ \frac{1}{2} \phi^*_{ab} (-\partial_\mu \partial^{\mu}) \phi_{ab} + U_{\text{mic}}(\rho, \sigma) \\
+ \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) (\partial^{\mu} \omega^{\nu} - \partial^{\nu} \omega^{\mu}) + \frac{1}{2} m^2_\omega \omega_\mu \omega^\mu
\]

\[
\phi_{ab} = \begin{pmatrix} 
\frac{1}{\sqrt{2}}(\sigma + i\pi^0) & i\pi^- \\
\pi^+ & \frac{1}{\sqrt{2}}(\sigma - i\pi^0)
\end{pmatrix} \quad U_{\text{mic}}(\rho, \sigma) = \tilde{U}(\rho) - m^2_\pi f_\pi \sigma \\
\rho = \frac{1}{2}(\sigma^2 + \pi^2)
\]

**Effective potential** constructed to reproduce standard nuclear thermodynamics around equilibrium
**Chemical freeze-out** in baryonic matter at $T < 100$ MeV is **not** associated with (chiral) phase transition or rapid crossover.
**Neutron Star Scenarios**

**Tolman-Oppenheimer-Volkov equations**

\[
\frac{dP}{dr} = -\frac{G}{c^2} \frac{(M + 4\pi Pr^3)(E + P)}{r(r - GM/c^2)}
\]

\[
\frac{dM}{dr} = 4\pi r^2 \frac{E}{c^2}
\]
New constraints from NEUTRON STARS

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest\textsuperscript{1}, T. Pennucci\textsuperscript{2}, S. M. Ransom\textsuperscript{1}, M. S. E. Roberts\textsuperscript{3} & J. W. T. Hessels\textsuperscript{4,5}

direct measurement of neutron star mass from increase in travel time near companion J1614-2230
most edge-on binary pulsar known (89.17°) + massive white dwarf companion (0.5 M\textsubscript{sun})

heaviest neutron star with 1.97±0.04 M\textsubscript{sun}

Constraints from **neutron star observables**

A.W. Steiner, J. Lattimer, E.F. Brown

K. Hebeler, J. Lattimer, C. Pethick, A. Schwenk
PRL 105 (2010) 161102

realistic “nuclear” EoS
A. Akmal, V.R. Pandharipande, D.G. Ravenhall

**“Exotic” equations of state ruled out?**
NEUTRON STAR MATTER
Equation of State

Including new neutron star constraints plus Chiral Effective Field Theory at lower density
NEUTRON STAR Equation of State

- ChEFT \((n, p, e, \mu)\)
- PNJL \(G_v/G = 0.7 (d, u, e)\)
- PNJL \(G_v = 0 (d, u, e)\)

(2 versions of 2-flavor chiral quark models)

- “green belt” permitted by M(R) constraints
- PNJL with vector coupling
- PNJL without vector coupling
- realistic “conventional” nuclear EoS

NEUTRON STAR MATTER
Mass - Radius relation

- Conventional hadronic \((\text{baryonic} + \text{mesonic})\) degrees of freedom

- In-medium Chiral Effective Field Theory up to 3 loops (reproducing thermodynamics of normal nuclear matter) including beta equilibrium \(n \leftrightarrow p + e, \mu\)

\[\rho_0 = 0.16 \text{ fm}^{-3}\]
(density of normal nuclear matter)

\[M = 2M_0\]
\[R = 11.9 \text{ km}\]
NEUTRON STARS: MASS and RADIUS constraints

from: J. Trümper
Irsee Symposium 2012

HARD Equation of State required !!

1 Largest mass J1614 - 2230 (Demorest et al. 2010)
2 Maximum gravity XTE 1814 – 338 (Bhattacharyya et al. 2005)
3 Minimum radius RXJ1856 - 3754 (Trümper et al. 2004)
4 Radius, 90% confidence limits LMXB 47 Tuc (Heinke et al. 2006)
5 Largest spin frequency J1748 – 2446 (Hessels et al. 2006)
**NEUTRON STAR MATTER**

Equation of State

- In-medium **Chiral Effective Field Theory** up to 3 loops (reproducing thermodynamics of normal nuclear matter)

- **3-flavor PNJL** model at high densities (incl. *strange* quarks)

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**Figure:**

- **Green** and **blue** belts permitted by \( M(R) \) constraints

- **Chiral EFT** EoS

- **Coexistence region:** Gibbs conditions

- **Beta equilibrium:** \( n \leftrightarrow p + e, \mu \)

- **Charge conservation**

- **Quark-nuclear coexistence** occurs (if at all) at baryon densities \( \rho > 5 \rho_0 \)
NEUTRON STAR MATTER
Mass - Radius Relation and Composition

- In-medium **Chiral Effective Field Theory** + 3-flavor **PNJL** model

**Chiral EFT + PNJL**

- $G_v = 0.5 \, \text{G}$
- $G_v = 0$

- Strangeness: small admixtures of $\Lambda$ hyperons possible at $\rho > 3 \, \rho_0$
  - Strong repulsion

$M / M_\odot$ vs. $R [\text{km}]$

$\rho / \rho_0$ vs. $\rho_0$


Steiner et al.

Trümper

$\rho_i / \rho$

neutrons

quarks

protons

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Densities and Scales in Compressed Baryonic Matter

\[ \rho_B = 0.15 \text{ fm}^{-3} \]

**normal nuclear matter:** dilute

\[ \rho_B = 0.6 \text{ fm}^{-3} \]

**neutron star core matter:** compressed but not superdense

- recall: chiral (soliton) model of the nucleon
- compact baryonic core
  \[ \langle r^2 \rangle_B^{1/2} \approx 0.5 \text{ fm} \]


- mesonic cloud
  \[ \langle r^2 \rangle_{E, \text{isoscalar}}^{1/2} \approx 0.8 \text{ fm} \]

... treated properly in chiral EFT
SUMMARY

- **Low-energy QCD**
  - Spontaneously broken **chiral symmetry**
  - **Effective Field Theory** of weakly interacting **Nambu-Goldstone bosons**

- **Nuclear thermodynamics**: Fermi liquid ↔ interacting Fermi gas
  - Framework: In-medium **Chiral Effective Field Theory**
  - **No** indication of first order chiral phase transition in the range $\rho \leq 2\rho_0$, $T \leq 100$ MeV

- New **dense & cold matter** constraints from **neutron stars**:
  - Mass - radius relation; observation of two-solar-mass n-star
  - “**Conventional**” (non-exotic) **EoS works best**
    (nuclear effective field theory + advanced many-body methods)
  - Small admixtures of **strangeness** possible, but need repulsive short-range hyperon-nucleon and hyperon-hyperon interactions
The End

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