NUCLEAR CHIRAL THERMODYNAMICS and PHASES of QCD

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Prelude: QCD Phase Diagram (Concepts, Models, Problems)

Main Theme: Nuclear Chiral Thermodynamics
- QCD interface with nuclear physics: Chiral Effective Field Theory
- Nuclear Equation of State and QCD phase diagram
- Density and temperature dependence of the Chiral (Quark) Condensate

Outlook: New constraints from Neutron Stars
Part I: Prelude

QCD PHASE DIAGRAM

Visions & Facts
**QCD PHASE DIAGRAM**

( theorists’ vision )

- **Spontaneous Chiral Symmetry Breaking**
  - $\bar{q}q$ condensation

- **High Density:**
  - Color Super Conductivity

- **Hadron Phase:**
  - $\langle \bar{q}q \rangle \neq 0$

- **CSC Phases:**
  - $\langle qq \rangle \neq 0$

- **Quark – Gluon Phase**

- **Critical Point**

- **Cooper Pairing**

- **Temperature** $T$ [GeV]
  - $T_c$

- **Baryon Chemical Potential** $\mu_B$
  - 1 GeV

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QCD PHASE DIAGRAM

(reality ? facts ?)

Lattice QCD

\( \bar{q}q \) condensation

Constraints from Nuclear Physics

New constraints from Neutron Stars

\[ T \text{ [GeV]} \]

\( \mu_B \)

hadron phase \( \langle \bar{q}q \rangle \neq 0 \)

quark – gluon phase

\( \langle q \rangle \neq 0 \)

critical point

\( T_c \)

nuclear matter
The result is shown in Fig. 7. This figure displays in addition the separate contributions obtained in the nonlocal PNJL model considered here. The dashed lines show the chiral condensate of the Polyakov loop.

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**MODELING the QCD PHASE DIAGRAM**

Guiding principle: QCD symmetries and symmetry breaking patterns

- Spontaneously broken chiral symmetry $SU(N_f)_R \times SU(N_f)_L$
- Non-local PNJL model
- Centre $Z(3)$ of $SU(3)_c$ gauge group

- Chiral and deconfinement crossover transitions (3 flavor PNJL model)

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The orange band shows the confinement-deconfinement crossover transition as described by the Polyakov loop in the range $k_l < \Phi < k_n$. The dashed black line corresponds to the chiral crossover $\bar{\psi}\psi/\bar{\psi}\psi < k_{ch}$. The solid black line indicates the chiral first-order transition. The temperature scale is set by $T_c$. Including wavefunctionrenormalization effects requires a careful reassessment of chiral low-energy theorems. Pseudoscalar meson masses and corresponding decay constants at zero temperature have been derived. The results clearly show that the formalism incorporates fundamental chiral relations such as the Gell-Mann–Oakes–Renner and Goldberger–Treiman relations. In the three-flavor case, the inclusion of the 't Hooft-Kobayashi-Maskawa interaction leads to the correct mass splitting between the $\eta$ and the $\eta'$ meson. The PNJL thermodynamics has now been developed with systematic inclusion of the quark-quasiparticle renormalization factor $Z_b$. The temperature dependence of the chiral condensate and of the Polyakov loop has been calculated, indicating chiral and deconfinement crossover transitions. We have compared our results with recent lattice QCD computations. Finally, a quark chemical potential has been introduced that enables extensions to the finite-density region of the QCD phase diagram. The impact of the wavefunction renormalization factor $Z_b$ compared to previous calculations setting $Z_b \equiv 1$ is generally quite small over the whole relevant momentum range. This can be understood considering the gap equations at zero temperature, since $Z_b$ deviates significantly from unity only in the momentum range $p \gtrsim 1$ GeV, its effect does not contribute much to the relevant integrals because of its suppression by the integration measure. With inclusion of $Z_b$, the chiral and deconfinement crossover transitions tend to become smoother compared to our previous investigations. The flavor dependence of the deconfinement temperature scale is an important issue in lowering chiral transition temperatures in accordance with the tendency recently.

Fig. 4. Phase diagram of our nonlocal PNJL model without the $Z$ factor in $T - \theta$ plane. The dotted and the solid line mean the crossover, first-order phase transition, respectively.

Fig. 5. The $T$ dependence of the $\bar{\sigma}$ and $\text{Re} \Phi$. The dotted and solid lines are results with $T_0 = 270$ and $218$ MeV, respectively.
The entropy density decreases for quark number density, Fig. 9, if the pressure at
findings for the pressure these corrections become more
limit always from below. Without the
in this figure. The quark density approaches the SB-
}
Part II:
NUCLEAR
CHIRAL
THERMODYNAMICS
NUCLEAR MATTER and QCD PHASES

nuclei

Scales in nuclear matter:

- momentum scale: **Fermi momentum**
  \[ k_F \simeq 1.4 \text{ fm}^{-1} \sim 2m_{\pi} \]
- NN distance:
  \[ d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3 \text{ m}_{\pi}^{-1} \]
- energy per nucleon:
  \[ E/A \simeq -16 \text{ MeV} \]
- compression modulus:
  \[ K = (260 \pm 30) \text{ MeV} \sim 2m_{\pi} \]
PIONS and NUCLEI in the context of LOW-ENERGY QCD

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**

LOW-ENERGY / LOW-TEMPERATURE QCD: Effective Field Theory of weakly interacting

Nambu-Goldstone Bosons (PIONS) representing QCD at (energy and momentum) scales

\[ Q << 4\pi f_\pi \sim 1 \text{ GeV} \]

\[ f_\pi = 92.4 \text{ MeV} \]

\[ m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi} \psi \rangle + O(m_q^2) \]

spontaneous symmetry breaking

explicit symmetry breaking
CHIRAL EFFECTIVE FIELD THEORY

- Systematic framework at interface of QCD and Nuclear Physics
- Interacting systems of PIONS (light / fast) and NUCLEONS (heavy / slow):

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, ...) \]

\[ U(x) = \exp[i\tau_a \pi_a(x)/f_\pi] \]

- Construction of Effective Lagrangian: Symmetries

  - short distance dynamics: contact terms
Explicit $\Delta(1230)$ DEGREES of FREEDOM

- **Large spin-isospin polarizability** of the Nucleon

  example: polarized Compton scattering

$$\beta_\Delta = \frac{g_A^2}{f^2_\pi(M_\Delta - M_N)} \sim 5 \text{ fm}^3$$

$$M_\Delta - M_N \simeq 2 m_\pi << 4\pi f_\pi$$

(small scale)

- **Pionic Van der Waals** - type intermediate range central potential


  N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259

  

  J. Fujita, H. Miyazawa (1957)

  Pieper, Pandharipande, Wiringa, Carlson (2001)

  strong 3-body interaction
Important pieces of the CHIRAL NUCLEON-NUCLEON INTERACTION

ISOVECTOR TENSOR FORCE

- $S_1 \rightarrow V_T \rightarrow S_2$

- Note: no $\rho$ meson

CENTRAL ATTRACTION from TWO-PION EXCHANGE

- $\Delta(1232)$
- Note: no $\sigma$ boson

Van der WAALS - like force:

$$V_c(r) \propto -\frac{\exp[-2m_\pi r]}{r^6}P(m_\pi r)$$

... at intermediate and long distance
**CHIRAL DYNAMICS and the NUCLEAR MANY-BODY PROBLEM**


- **Small scales:**
  \[ k_F \sim 2m_\pi \sim M_\Delta - M_N < < 4\pi f_\pi \]

- **PIONS** (and **DELTA** isobars) as *explicit degrees of freedom*

**IN-MEDIUM CHIRAL PERTURBATION THEORY**

Pion exchange processes in presence of filled *Fermi sea*

\[ \pi + N + N \rightarrow \pi + N + N + \ldots \]

2nd order **TENSOR** force + nucleon’s **SPIN-ISOSPIN** polarizability

Short-distance dynamics: \[ N \times N \]

Contact interactions (incl. resummations)
IN-MEDIUM CHIRAL PERTURBATION THEORY

- **Loop expansion** of (In-Medium) Chiral Perturbation Theory
  
  Systematic expansion of ENERGY DENSITY $\mathcal{E}(k_F)$ in powers of Fermi momentum [modulo functions $f_n(k_F/m_\pi)$]
  
  (works for $k_F < < 4\pi f_\pi \sim 1$ GeV)

- **Finite nuclei ↔ energy density functional**
  

  many quantitatively successful applications throughout the nuclear chart

  e.g. P. Finelli et al.: Nucl. Phys. A 770 (2007) 1

- **Nuclear thermodynamics:** compute free energy density

  (3-loop order)


  in-medium nucleon propagators incl. Pauli blocking
In-medium ChPT
3-loop \((\pi, N, \Delta)\)

Input parameters:
two contact terms

basically:
analytic calculation

Output:

- Binding & saturation
  \[ \frac{E_0}{A} = -16 \text{ MeV} \quad \rho_0 = 0.16 \text{ fm}^{-3} \quad K = 290 \text{ MeV} \]

- Realistic (complex, momentum dependent) single-particle potential
  ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)

- Asymmetry energy
  \[ A(k^0_F) = 34 \text{ MeV} \]

- Landau parameters
  \[ J.W. Holt, N. Kaiser, W.W. \]
  \[ arXiv:1106.5702 [nucl-th], NPA (2011) \]
NUCLEAR THERMODYNAMICS

NUCLEAR CHIRAL (PION) DYNAMICS

BINDING & SATURATION:

Van der Waals + Pauli

\[ \pi \]

\[ N, \Delta \]

3-body forces

contact terms

Liquid - Gas Transition at

Critical Temperature $T_c = 15$ MeV

(empirical: $T_c = 16 - 18$ MeV)

NUCLEAR THERMODYNAMICS

Skryme phenomenology

G. Sauer, H. Chandra, U. Mosel
Nucl. Phys. A 264 (1976) 221

Multifragmentation and fission analysis

V.A. Karnaukhov et al. :
**PHASE DIAGRAM of NUCLEAR MATTER**

- **In-medium chiral effective field theory**
  (3-loop calculation of free energy density)


- Pion-nucleon dynamics incl. delta isobars
- Short-distance NN contact terms
- Three-body forces
PHASE DIAGRAM of NUCLEAR MATTER

Trajectory of CRITICAL POINT for asymmetric matter as function of proton fraction $Z/A$

...determined almost entirely by isospin dependent (one- and two-) pion exchange dynamics

**CHIRAL CONDENSATE at finite BARYON DENSITY**

- Chiral (quark) condensate $\langle \bar{q}q \rangle$:
  
  \[ m_\pi^2 f_\pi^2 = -2 m_q \langle \bar{q}q \rangle \]

  Order parameter of spontaneously broken chiral symmetry in QCD

- Hellmann - Feynman theorem: 
  
  \[ \langle \Psi | \bar{q}q | \Psi \rangle = \langle \Psi \\frac{\partial H_{\text{QCD}}}{\partial m_q} | \Psi \rangle = \frac{\partial \mathcal{E}(m_q; \rho)}{\partial m_q} \]

- Sigma term

\[ m_q \frac{\partial M_N}{\partial m_q} \]

- In-medium chiral effective field theory

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{f_\pi^2} \left[ \frac{\sigma_N}{m_\pi^2} \left( 1 - \frac{3 p_F^2}{10 M_N^2} + \ldots \right) + \frac{\partial}{\partial m_\pi^2} \left( \frac{E_{\text{int}}(p_F)}{A} \right) \right] \]

- (free) Fermi gas of nucleons

- Nuclear interactions (dependence on pion mass)
CHIRAL CONDENSATE: DENSITY DEPENDENCE

- **In-medium Chiral Effective Field Theory**
  
  (NLO 3-loop)

  constrained by realistic nuclear equation of state

  N. Kaiser, Ph. de Homont, W.W.

- **Substantial change of symmetry breaking scenario**
  between chiral limit $m_q = 0$ and physical quark mass $m_q \sim 5$ MeV

- **Nuclear Physics** would be **very different** in the **chiral limit**!
CHIRAL CONDENSATE: 
DENSITY and TEMPERATURE DEPENDENCE

- Free energy density 
  \[ \mathcal{F}(m_q; \rho, T) \]

- In-medium Chiral Effective Field Theory 
  (NLO 3-loop)

  constrained by realistic nuclear equation of state

- No indication of first order chiral phase transition for 
  \[ \rho \lesssim 2 \rho_0 , \quad T \lesssim 100 \text{ MeV} \]

\[ \langle \Psi | \bar{q}q | \Psi \rangle_{\rho, T} = \frac{\partial \mathcal{F}(m_q; \rho, T)}{\partial m_q} \]

\[ \frac{\langle \bar{q}q \rangle_{\rho, T}}{\langle \bar{q}q \rangle_0} \]

S. Fiorilla, N. Kaiser, W.W.
arXiv:1104.2819 [nucl-th]

symmetric nuclear matter 
\[ N = Z \]

\[ T = 0 \quad 20 \quad 50 \quad T = 100 \text{ MeV} \]
Liquid-gas phase transition leaves its signature also in chiral condensate but: no tendency toward chiral first order transition in the range $\mu_B \lesssim 1$ GeV.
Major challenge: design QCD phase diagram in accordance with known realistic features from hadronic and nuclear physics.
Outlook:

New Constraints from NEUTRON STARS
A two-solar-mass neutron star measured using Shapiro delay

P. B. Demoerst, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels

Direct measurement of neutron star mass from increase in travel time near companion J1614-2230 most edge-on binary pulsar known (89.17°) + massive white dwarf companion (0.5 $M_{\text{sun}}$)

Heaviest neutron star with 1.97±0.04 $M_{\text{sun}}$
TWO-SOLAR-MASS NEUTRON STAR

... measured using Shapiro delay

P.B. Demorest et al., Nature 467 (2010) 1081
News from NEUTRON STARS

K. Hebeler, J. Lattimer, C. Pethick, A. Schwenk
PRL 105 (2010) 161102

New constraints from EFT and neutron star observables

“Exotic” equations of state ruled out?

realistic “nuclear” EoS (Illinois)

A.W. Steiner, J. Lattimer, E.F. Brown
Including new neutron star constraints plus Chiral Effective Field Theory at lower density
SUMMARY

- Exploration of **QCD phase diagram**: progress concerning basic symmetry breaking patterns
  - Lattice QCD (restricted to small quark chemical potentials)
  - Models (PNJL, PQM) (but: nuclear physics constraints missing)
  - Dyson-Schwinger QCD ( -- same problem -- )

- **Nuclear thermodynamics** based on In-medium **Chiral Effective Field Theory**
  - Fermi liquid ↔ interacting Fermi gas (1st order transition)
  - **No** indication of first order **chiral** transition in the range
    \[ \rho \lesssim 2 \rho_0 , \ T \lesssim 100 \text{MeV} \]
  - Major challenge: design **QCD phase diagram** that is consistent with established hadronic and nuclear physics

- New **dense & cold matter** constraints from **neutron stars**:
  - Mass - radius relation; observation of two-solar-mass n-star
  - “Non-exotic” equation of state works best!
The End

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