Theory of ANTIIKAI\textsc{on} interactions with NUCLE\textsc{ons} and NUCLE\textsc{I}

a state-of-the-art report

Wolfram Weise
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- Low-energy QCD: symmetries and symmetry breaking patterns
- Strangeness and chiral SU(3) dynamics
- $\bar{K}N$ threshold physics and kaonic hydrogen
- Antikaons in baryonic matter
- New constraints from neutron stars
Hierarchy of **QUARK MASSES** in **QCD**

**“light” quarks**

\[ m_d \approx 4 - 6 \text{ MeV} \]
\[ m_u/m_d \approx 0.3 - 0.6 \]
\[ m_s \approx 80 - 130 \text{ MeV} \]
\[ (\mu \approx 2 \text{ GeV}) \]

**LOW-ENERGY QCD: CHIRAL EFFECTIVE FIELD THEORY**

- expansion in \( m_q \)
- and in powers of low momentum

**“heavy” quarks**

\[ m_c \approx 1.25 \text{ GeV} \]
\[ m_b \approx 4.2 \text{ GeV} \]
\[ m_t \approx 174 \text{ GeV} \]

**Non-Relativistic QCD: HEAVY QUARK EFFECTIVE THEORY**

- expansion in powers of \( 1/m_Q \)
LOW-ENERGY QCD
with
STRANGE QUARKS:

Chiral SU(3) Dynamics

... realized as an EFFECTIVE FIELD THEORY with SU(3) octet of pseudoscalar Nambu-Goldstone bosons coupled to the baryon octet.

... explicit chiral symmetry breaking by non-zero quark masses (at a renormalization scale $\mu \sim 2$ GeV):

$$m_q = \frac{m_u + m_d}{2} = 3 - 5 \text{ MeV} \quad m_s \sim 25 \, m_q$$
Strange quarks are intermediate between “light” and “heavy”:
- interplay between spontaneous and explicit chiral symmetry breaking in low-energy QCD
- Testing ground: high-precision antikaon-nucleon threshold physics
  - strongly attractive low-energy $\bar{K}N$ interaction
- Nature and structure of $\Lambda(1405)$ ($B = 1$, $S = -1$, $J^P = 1/2^-$)
  - three-quark valence structure vs. “molecular” meson-baryon system?
- Quest for quasi-bound antikaon-nuclear systems?
- Role of strangeness in dense baryonic matter?
  - new constraints from neutron stars
NAMBU - GOLDSTONE BOSONS:
Pseudoscalar SU(3) meson octet
\[ \{\phi_a\} = \{\pi, K, \bar{K}, \eta_8\} \]

ORDER PARAMETERS:
\[ \langle 0 | A_{\mu}^a(0) | \phi_b(p) \rangle = i\delta_{ab} p^\mu f_b \]

DECAY CONSTANTS
(chiral limit: \( f = 86.2 \) MeV)
\[ f_\pi = 92.4 \pm 0.3 \text{ MeV} \]
\[ f_K = 110.0 \pm 0.9 \text{ MeV} \]
\[ f_\eta = 120.1 \pm 4.6 \text{ MeV} \]

Gell-Mann Oakes Renner relations
\[ m_\pi^2 f_\pi^2 = -\frac{m_u + m_d}{2} \langle \bar{u}u + \bar{d}d \rangle \]
\[ m_K^2 f_K^2 = -\frac{m_u + m_s}{2} \langle \bar{u}u + \bar{s}s \rangle + \text{higher order corrections} \]
CHIRAL SU(3) EFFECTIVE FIELD THEORY

- Interacting systems of **NAMBU-GOLDSTONE BOSONS** (pions, kaons) coupled to **BARYONS**

\[ \mathcal{L}_{eff} = \mathcal{L}_{mesons}(\Phi) + \mathcal{L}_B(\Phi, \Psi_B) \]

- Leading **DERIVATIVE** couplings (involving \( \partial^\mu \Phi \))
  determined by spontaneously broken **CHIRAL SYMMETRY**

- **Low-Energy Expansion:** **CHIRAL PERTURBATION THEORY**
  
  "small parameter": \( \frac{p}{4\pi f_\pi} \sim \frac{\text{energy / momentum}}{1 \text{ GeV}} \)

- works well for low-energy **pion-pion** and **pion-nucleon** interactions

- ... but **NOT** for systems with **strangeness** \( S = -1 \) (\( \bar{K}N \), \( \pi\Sigma \), ...)

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**Low-Energy $\bar{K}N$ Interactions**

- Chiral Perturbation Theory **NOT** applicable: $\Lambda(1405)$ resonance 27 MeV below $K^-p$ threshold

  $\Sigma^*(1385) \quad \Lambda^*(1405)$

  ![Diagram](image)

  Non-perturbative **Coupled Channels** approach based on **Chiral SU(3) Dynamics**


- Leading $s$-wave $l = 0$ meson-baryon interactions (Tomozawa-Weinberg)

  ![Diagram](image)
**CHIRAL SU(3) COUPLED CHANNELS DYNAMICS**

\[
T_{ij} = K_{ij} + \sum_n K_{in} G_n T_{nj}
\]

- Leading s-wave \( I = 0 \) meson-baryon interactions (Tomozawa-Weinberg)
  
  Note: **ENERGY DEPENDENCE** characteristic of Nambu-Goldstone Bosons

\[
|1\rangle = |\bar{K}N, I = 0\rangle \\
|2\rangle = |\pi\Sigma, I = 0\rangle
\]

\[
\begin{align*}
K_{11} &= \frac{3}{2 f_K^2} (\sqrt{s} - M_N) \\
K_{22} &= \frac{2}{f_\pi^2} (\sqrt{s} - M_\Sigma)
\end{align*}
\]

- driving interactions individually **strong** enough to produce
  
  - \( \bar{K}N \) **bound state**
  - \( \pi\Sigma \) **resonance**

- **strong** channel coupling
  
  \(12 \leftrightarrow 21:\)

\[
\begin{align*}
K_{12} &= \frac{-1}{2 f_\pi f_K} \sqrt{3} \left(\sqrt{s} - \frac{M_\Sigma + M_N}{2}\right)
\end{align*}
\]
CHIRAL SU(3) COUPLED CHANNELS DYNAMICS

\[ T_{ij} = K_{ij} + \sum_n K_{in} G_n T_{nj} \]

- **input from**
  - chiral SU(3) meson-baryon effective Lagrangian

- **loop functions**
  - (dim. regularization)
  - with subtraction constants
  - encoding short distance dynamics

**coupled channels:**

- \( K^- p, \quad K^0 n, \quad \pi^0 \Sigma^0, \quad \pi^+ \Sigma^-, \quad \pi^- \Sigma^+, \quad \pi^0 \Lambda, \quad \eta \Lambda, \quad \eta \Sigma^0, \quad K^+ \Xi^-, \quad K^- \Xi^0 \)
**CHIRAL SU(3) COUPLED CHANNELS DYNAMICS:**
- NLO hierarchy of driving terms -

**Input:**
- Physical pion and kaon decay constants
- Axial vector constants
- D and F from hyperon beta decays
- 7 low-energy constants

**Leading Order (Weinberg-Tomozawa) Terms**

\[
\mathcal{L}^{MB}_1 = \text{Tr} \left( \frac{D}{2} (\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}) + \frac{F}{2} (\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]) \right)
\]

**Direct and Crossed Born Terms**

\[
g_A = D + F = 1.26
\]

\[
\mathcal{L}^{MB}_2 = b_D \text{Tr}(\bar{B}\{\chi_+, B\}) + b_F \text{Tr}(\bar{B}[\chi_+, B]) + b_0 \text{Tr}(\bar{B}B)\text{Tr}(\chi_+)
\]

\[
+ d_1 \text{Tr}(\bar{B}\{u^\mu, [u_\mu, B]\}) + d_2 \text{Tr}(\bar{B}[u^\mu, [u_\mu, B]])
\]

\[
+ d_3 \text{Tr}(\bar{B}u_\mu)\text{Tr}(u^\mu B) + d_4 \text{Tr}(\bar{B}B)\text{Tr}(u^\mu u_\mu),
\]

**Next-to-Leading Order (NLO)**

\[\mathcal{O}(p^2)\]
The TWO POLES scenario

\[ \Lambda(1405) \]

\[ |T| \quad [\text{MeV}^{-1}] \]

\[ \text{Re}[z] \quad [\text{MeV}] \]

\[ \text{Im}[z] \quad [\text{MeV}] \]


**Kaonic hydrogen** precision data

**strong interaction shift and width:**

\[ \Delta E = 283 \pm 36 \text{ (stat)} \pm 6 \text{ (syst)} \text{ eV} \]

\[ \Gamma = 541 \pm 89 \text{ (stat)} \pm 22 \text{ (syst)} \text{ eV} \]

**Theory:**

leading order

(Tomozawa - Weinberg)

B. Borasoy, R. Nißler, W.W.


B. Borasoy, U.-G. Meißner, R. Nißler

PRC74 (2006) 055201

Technische Universität München
Improved constraints on chiral SU(3) dynamics from kaonic hydrogen

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Abstract

A new improved study of $K^{-}$-proton interactions near threshold is performed using coupled-channels dynamics based on the next-to-leading order chiral SU(3) meson-baryon effective Lagrangian. Accurate constraints are now provided by new high-precision kaonic hydrogen measurements. Together with threshold branching ratios and scattering data, these constraints permit an updated analysis of the complex $\bar{K}N$ and $\pi\Sigma$ coupled-channels amplitudes and an improved determination of the $K^{-}p$ scattering length, including uncertainty estimates.
UPATED ANALYSIS of $K^- p$ THRESHOLD PHYSICS


- Chiral SU(3) coupled-channels dynamics
  Tomozawa-Weinberg + Born terms + NLO

<table>
<thead>
<tr>
<th>kaonic hydrogen shift &amp; width</th>
<th>theory (NLO)</th>
<th>exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E$ (eV)</td>
<td>306</td>
<td>283 ± 36 ± 6</td>
</tr>
<tr>
<td>$\Gamma$ (eV)</td>
<td>591</td>
<td>541 ± 89 ± 22</td>
</tr>
</tbody>
</table>

threshold branching ratios

| $\Gamma(K^- p \rightarrow \pi^+ \Sigma^-)/\Gamma(K^- p \rightarrow \pi^- \Sigma^+)$ | 2.37 | 2.36 ± 0.04 |
| $\Gamma(K^- p \rightarrow \pi^+ \Sigma^-, \pi^- \Sigma^+)/\Gamma(K^- p \rightarrow \text{all inelastic channels})$ | 0.66 | 0.66 ± 0.01 |
| $\Gamma(K^- p \rightarrow \pi^0 \Lambda)/\Gamma(K^- p \rightarrow \text{neutral states})$ | 0.19 | 0.19 ± 0.02 |

scatterings length (fm)

Re $a(K^- p) = -0.65 ± 0.10$  Im $a(K^- p) = 0.81 ± 0.15$

best fit achieved with $\chi^2/d.o.f. \approx 0.9$
**Non-trivial result:**
best NLO fit prefers **physical** values of **decay constants**:

<table>
<thead>
<tr>
<th>Decay Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_K ) (MeV)</td>
<td>110.0</td>
</tr>
<tr>
<td>( f_\eta ) (MeV)</td>
<td>118.8</td>
</tr>
</tbody>
</table>

\( (f_\pi = 92.4 \text{ MeV}) \)

- **Tomozawa-Weinberg terms dominant**
- **Born terms significant**
- **NLO parameters are non-negligible but small**
- Subtraction constants (encoding unresolved **high energy** behaviour) are of “natural” size
UPDATED ANALYSIS of $K^-p$ LOW-ENERGY CROSS SECTIONS

$\sigma(K^-p \rightarrow K^-p)$ [mb]

$\sigma(K^-p \rightarrow \pi^+\Sigma^-)$ [mb]

$\sigma(K^-p \rightarrow \pi^-\Sigma^+)$ [mb]

$\sigma(K^-p \rightarrow \pi^0\Sigma^0)$ [mb]
**K^-p SCATTERING AMPLITUDE**

\[
f(K^-p) = \frac{1}{2} \left[ f_{\bar{K}N}(I = 0) + f_{\bar{K}N}(I = 1) \right]
\]

threshold region and subthreshold extrapolation:

\[
\Lambda(1405): \quad \bar{K}N \ (I = 0) \; \text{quasibound state embedded in the } \pi \Sigma \; \text{continuum}
\]

---

**Complex scattering length (including Coulomb corrections)**

\[
\text{Re } a(K^-p) = -0.65 \pm 0.10 \quad \text{fm}
\]

\[
\text{Im } a(K^-p) = 0.81 \pm 0.15 \quad \text{fm}
\]

The $K^-n$ scattering length is calculated as:

$$a_{K^-n} = 0.29 + 0.76i \text{ fm (WT)}$$

$$a_{K^-n} = 0.27 + 0.74i \text{ fm (WTB)}$$

$$a_{K^-n} = 0.57 + 0.72i \text{ fm (NLO)}$$

The scattering amplitude is shown in Fig. 1. The jump of the real part of the scattering length in the step WTB → NLO is correlated with the jump of the $K^-p$ scattering length:

$$a_{K^-p} = -0.93 + 0.82i \text{ fm (WT)}$$

$$a_{K^-p} = -0.94 + 0.85i \text{ fm (WTB)}$$

$$a_{K^-p} = -0.70 + 0.89i \text{ fm (NLO)}$$

Note that the results with the WT and WTB models are a bit off the SIDDHARTA result.

**Figure 1:** Scattering amplitude in $K^-n$ channel.
Implications & Comments

- $K^- p$ scattering length more accurately determined than $K^- n$
  (SIDDHARTA constraints vs. uncertainties in $I = 1$ channels)

- **Kaonic deuterium** measurements important for providing further constraints on $K^- n$ interaction

- $B = 2$ systems - key issue:
  \[ \bar{K}NN \rightarrow YN \]
  absorption into non-mesonic hyperon - nucleon final states
  e.g.:
  \[
  \begin{array}{c}
  \text{p} \\
  \text{K}^- \\
  \text{n}
  \end{array}
  \quad \quad \quad
  \begin{array}{c}
  \text{Λ}(1405) \\
  \pi \\
  \text{n}
  \end{array}
  \quad \quad \quad
  \begin{array}{c}
  \Sigma \\
  \text{N}
  \end{array}
  \]

  **Repulsive short-distance** $\Lambda^* (uds) N$ interaction?

- **$\Lambda^* -$ nucleon potential**

```
\[
\Lambda^* \rightarrow \text{Nucleon potential}
\]
```

- Lattice QCD
  Y. Ikeda et al. (HAL QCD collaboration)
Kaons and Antikaons in Nuclear Matter

In-medium Chiral SU(3) Dynamics with Coupled Channels

Kaon spectrum in baryonic matter determined by:

\[
\omega^2 - \vec{q}^2 - m_K^2 - \Pi_K(\omega, \vec{q}; \rho) = 0
\]

\[
\Pi_K^{-} = 2\omega U_K^{-} = -4\pi \left[ f_{K^-p} \rho_p + f_{K^-n} \rho_n \right] + \ldots +
\]

Pauli blocking, Fermi motion, 2N correlations

Note: In-medium \( \bar{K} \) width drops when mass falls below \( \pi \Sigma \) threshold
first suggested by D. Kaplan, A. Nelson (1985) on the basis of attractive $\bar{K}N$ Tomozawa - Weinberg term

at high density, energetically favourable to condense $K^-$

conversion to hyperons via $K^\text{-}NN \rightarrow YN$
Outlook: new constraints from NEUTRON STARS

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest\(^1\), T. Pennucci\(^2\), S. M. Ransom\(^1\), M. S. E. Roberts\(^3\) & J. W. T. Hessels\(^4,5\)


direct measurement of neutron star mass from increase in travel time near companion J1614-2230
most edge-on binary pulsar known (89.17°) + massive white dwarf companion (0.5 \(M_{\text{sun}}\))

heaviest neutron star with 1.97±0.04 \(M_{\text{sun}}\)
**News from NEUTRON STARS**

K. Hebeler, J. Lattimer, C. Pethick, A. Schwenk
PRL 105 (2010) 161102

A.W. Steiner, J. Lattimer, E.F. Brown

### Realistic “nuclear” EoS

A. Akmal, V.J. Pandharipande, D.G. Ravenhall

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- **New constraints from EFT and neutron star observables**
- **kaon condensate**
- **quark matter**
- **“Exotic” equations of state ruled out?**
NEUTRON STAR MATTER
Equation of State

Low-density (crust) + ChEFT (FKW)
Constrained extrapolation (polytropes)
Akmal, Pandharipande, Ravenhall (1998)

\[ P = \text{const} \cdot \rho^\Gamma \]

Including new neutron star constraints plus Chiral Effective Field Theory at lower density

S. Fiorilla, N. Kaiser, W.W.
Nucl. Phys.
A 880 (2012) 65

B. Röttgers, W.W.
(2011)

W.W.
Prog. Part. Nucl. Phys.
(2012), in print
**NEUTRON STAR MATTER**

Mass - Radius relation

- **Option I:**
  - Conventional hadronic (*baryonic + mesonic*) degrees of freedom
  - In-medium Chiral Effective Field Theory up to 3 loops
    (reproducing thermodynamics of normal nuclear matter)


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**Graphs:**

- **Left graph:**
  - Mass ($M / M_\odot$) vs. Radius ($R [\text{km}]$)
  - "conventional" eq. of state nucleons + pions $\Delta(1230)$ 3 body forces
  - PSR J1614-2230

- **Right graph:**
  - Density profile
  - $M = 2.00 M_\odot$
  - $R = 11.94 \text{ km}$

Option II:
Polyakov - Nambu - Jona-Lasinio (PNJL) model
(u-,d- and s-quarks as quasiparticles with dynamically generated constituent masses)

... features **first order chiral phase transition**

at low temperatures and moderate baryon chemical potentials

... produces **too soft** equation of state for neutron matter → does not work

Option III:
Conventional hadronic (ChEFT) EoS

matched smoothly at densities

\[ \rho \simeq 3 \rho_0 \]

to PNJL EoS including repulsive vector coupling between quarks

**no 1st order phase transition**

soft **crossover** between hadronc and quark phases
**SUMMARY**

- **New** consistent analysis of $\bar{K}N$ threshold physics and scattering data based on chiral SU(3) effective Lagrangian at next-to-leading order.

- **New** evaluation of $K^- p$ scattering length:
  
  \[
  a(K^- p) = -0.65 + 0.81 \text{ i [fm]} \quad (\sim 15 \% \text{ accuracy})
  \]

  deduced:
  
  \[
  a(K^- n) \simeq 0.6 + 0.7 \text{ i [fm]} \quad (\text{less accurate})
  \]

- Need kaonic deuterium to complete $\bar{K}N$ and set constraints for $\bar{K}NN \rightarrow YN$ absorption channel.

- **New** constraints from two-solar-mass neutron star and window of n-star radii: 
  
  *conventional* EoS works best - *kaon condensate* ruled out 
  
  *(hyperons* may contribute *if* short-range *YN* interactions sufficiently repulsive).