CHIRAL DYNAMICS Realizations of QCD in HADRONIC and NUCLEAR PHYSICS

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- **Nuclear chiral dynamics**
  QCD interface of nuclear physics: **Chiral Effective Field Theory**

- **Pions** and tensor force in the nuclear many-body problem

- Nuclear thermodynamics

- Density dependence of **chiral** (quark) **condensate**
1 Prelude: **PHASES and STRUCTURES of QCD**

**QCD PHASE DIAGRAM**

- **nuclei**

**Scales in nuclear matter**

- momentum scale: **Fermi momentum**
  \[ k_F \simeq 1.4 \text{ fm}^{-1} \simeq 2m_\pi \]

- NN distance:
  \[ d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3 \text{ m}_\pi^{-1} \]

- energy per nucleon:
  \[ E/A \simeq -16 \text{ MeV} \]

- compression modulus:
  \[ K = (260 \pm 30) \text{ MeV} \]
Spontaneously Broken CHIRAL SYMMETRY

- NAMBU - GOLDSTONE BOSON: PION

- ORDER PARAMETER: PION DECAY CONSTANT

\[ \langle 0 | A_{\mu}^a(0) | \pi^b(p) \rangle = i \delta^{ab} p_\mu f_\pi \]

Axial current

\[ f_\pi = 92.4 \text{ MeV} \]

- SYMMETRY BREAKING SCALE:

\[ \Lambda_\chi = 4\pi f_\pi \sim 1 \text{ GeV} \]

- PCAC:

\[ m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi} \psi \rangle + O(m_q^2) \]

Gell-Mann - Oakes - Renner Relation
SYMMETRY BREAKING PATTERN

PSEUDOSCALAR MESON SPECTRUM

mass [GeV]

0

SU(3)_L × SU(3)_R

π, K, η_8

m_0 = 0

m_s ≃ 130 MeV

m_u,d ≃ 5 MeV

U(1)_A breaking

η

η_0

η'

calculations:

Nambu & Jona-Lasinio model with N_f = 3 quark flavors

Nucl. Phys. A 516 (1990) 429

T. Hatsuda, T. Kunihiro
Phys. Reports 247 (1994) 221
**CHIRAL EFFECTIVE FIELD THEORY**

Gasser & Leutwyler   Weinberg   Ecker   ... many others

**LOW-ENERGY QCD:** **Effective Field Theory** of weakly interacting **Nambu-Goldstone Bosons** (**PIONS**) representing QCD at scales $Q \ll 4\pi f_\pi \sim 1\text{ GeV}$

- **PIONS** (light / fast) and **NUCLEONS** (heavy / slow):

  $$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, ...)$$
  $$U(x) = \exp[i\tau_a \pi_a(x)/f_\pi]$$

- Construction of Effective Lagrangian: **Symmetries**

  short distance dynamics: **contact terms**

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Low-Energy Expansion: CHIRAL PERTURBATION THEORY

- small parameter:

\[
\frac{Q}{4\pi f_\pi} \quad \text{energy / momentum / pion mass / 1 GeV}
\]

successfully applied to:

- PION-PION scattering
- PION-NUCLEON scattering
- PION photoproduction and COMPTON scattering on the NUCLEON
- long range NUCLEON-NUCLEON interaction
- NUCLEAR MATTER and NUCLEI
2
Nuclear Forces
- Recent Developments -

Hierarchy of SCALES

Early history:
M. Taketani,
S. Nakamura,
M. Sasaki
6 (1951) 581

Contemporary approach:
Chiral Effective Field Theory +
Lattice QCD
**NUCLEAR INTERACTIONS** from** CHIRAL EFFECTIVE FIELD THEORY**

Weinberg  Bedaque & van Kolck  Bernard, Epelbaum, Kaiser, Meißner; ... 

<table>
<thead>
<tr>
<th>( \mathcal{O} \left( \frac{Q^0}{\Lambda^0} \right) )</th>
<th>Two-nucleon force</th>
<th>Three-nucleon force</th>
<th>Four-nucleon force</th>
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<tbody>
<tr>
<td>LO</td>
<td><img src="image" alt="Diagram" /></td>
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<tr>
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**Systematically organized HIERARCHY**
NN Scattering Phase Shifts
from CHIRAL EFFECTIVE FIELD THEORY


quantitatively accurate at same level of precision as best phenomenological potentials
CHIRAL EFFECTIVE FIELD THEORY
at work in nuclear few-body systems

- example: elastic $\text{nd}$ scattering

**differential cross sections [mb/sr]**

- $\frac{d\sigma}{d\Omega}$
  - $3\text{ MeV}$
  - $10\text{ MeV}$
  - $65\text{ MeV}$

**vector analyzing powers**

- $A_y$
  - $3\text{ MeV}$
  - $10\text{ MeV}$

**tensor analyzing powers**

- $iT_{11}$
  - $N^2\text{LO}$
  - $10\text{ MeV}$
- $T_{20}$
  - $N^2\text{LO}$
- $T_{21}$
- $T_{22}$

Explicit $\Delta(1230)$ DEGREES of FREEDOM

Large spin-isospin polarizability of the Nucleon

example: polarized Compton scattering

$$\beta_\Delta = \frac{g_A^2}{f_\pi^2(M_\Delta - M_N)} \sim 5 \text{ fm}^3$$

$$M_\Delta - M_N \simeq 2 m_\pi << 4\pi f_\pi$$

(small scale)

Pionic Van der Waals - type intermediate range central potential

N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259

$V_c(r) = -\frac{9 g_A^2}{32\pi^2 f_\pi^2} \beta_\Delta \frac{e^{-2m_\pi r}}{r^6} P(m_\pi r)$

strong 3-body interaction

N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259

J. Fujita, H. Miyazawa; Prog. Theor. Phys. 17 (1957) 360
Pieper, Pandharipande, Wiringa, Carlson, PRC64 (2001) 014001
**Explicit $\Delta(1230)$ DEGREES of FREEDOM (contd.)**

<table>
<thead>
<tr>
<th>standard chiral EFT</th>
<th>Including $\Delta$ as an explicit DOF</th>
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<tr>
<td>LO</td>
<td>$X \hspace{1cm} H$</td>
</tr>
<tr>
<td>NLO</td>
<td>$X \hspace{1cm} \text{diagrams}$ + $h_A$</td>
</tr>
<tr>
<td>$N^2$LO</td>
<td>$\text{diagrams}$ + $b_3 + b_8$</td>
</tr>
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</table>

- **Important physics** of $\Delta(1230)$ promoted to **NLO**
- **Improved** convergence

Kaiser et al., Ordonez et al. (2007)

Krebs, Epelbaum, Meißner (2007)
Important pieces of the CHIRAL NUCLEON-NUCLEON INTERACTION

**ISOVECTOR TENSOR FORCE**

$S_1 \xrightarrow{V_T} S_2$

- note: **no** $\rho$ meson

**CENTRAL ATTRACTION** from **TWO-PION EXCHANGE**

$\Delta(1232)$

- note: **no** fictitious $\sigma$ boson

Van der WAALS - like force:

$$V_c(r) \propto -\frac{\exp[-2m_\pi r]}{r^6}P(m_\pi r)$$

... at intermediate and long distance

Short distance: **NN POTENTIAL from LATTICE QCD**


Reconstruct potential from wave function:

$V_C(r) = E + \frac{\nabla^2 \phi(r)}{2\mu} \phi(r)$

**Repulsive core from Lattice QCD**
PIONS (and DELTA isobars) as explicit degrees of freedom

Small scales:

\[ k_F \sim 2 m_\pi \sim M_\Delta - M_N \ll 4\pi f_\pi \]

IN-MEDIUM CHIRAL PERTURBATION THEORY

\[ \text{pion exchange processes in presence of filled Fermi sea} \]

2nd order TENSOR force + nucleon’s SPIN-ISOSPIN polarizability

short-distance dynamics: contact interactions
In-medium **nucleon propagator:**

\[
\frac{i}{\gamma \cdot p - M_N + i \epsilon} - 2 \pi (\gamma \cdot p + M_N) \delta(p^2 - M_N^2) \theta(p_0) \theta(k_F - |\vec{p}|)
\]

**Loop expansion** in ChPT

Systematic expansion of **ENERGY DENSITY** \( \mathcal{E}(k_F) \) in **powers of Fermi momentum** [modulo functions \( f_n(k_F/m_\pi) \)]

**Finite nuclei** \( \leftrightarrow \) energy **density functional**

**Nuclear thermodynamics:** compute **free energy density**

(3-loop order)


**in-medium** nucleon propagators incl. Pauli blocking
In-medium ChPT

3-loop \((\pi, N, \Delta)\)

Input parameter:

single contact term

basically:

analytic calculation

Output:

- Binding & saturation
  \[ E_0/A = -16 \text{ MeV} , \quad \rho_0 = 0.16 \text{ fm}^{-3} , \quad K = 290 \text{ MeV} \]

- Realistic (complex, momentum dependent) single-particle potential
  ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)

- Asymmetry energy \( A(k_F^0) = 34 \text{ MeV} \)
  - Landau parameters
NUCLEAR THERMODYNAMICS

NUCLEAR CHIRAL (PION) DYNAMICS

BINDING & SATURATION:
Yukawa + Van der Waals + Pauli

\[ V(r) \sim -\frac{e^{-2m_\pi r}}{r^6} P(m_\pi r) \]

+ 3-body forces + contact terms

nuclear matter: equation of state

pressure

3-loop in-medium ChEFT

\[ T = 25 \text{ MeV} \]
\[ T = 20 \text{ MeV} \]
\[ T = 15 \text{ MeV} \]
\[ T = 10 \text{ MeV} \]
\[ T = 5 \text{ MeV} \]

Critical Temperature \( T_c = 15 \text{ MeV} \)
(empirical: \( T_c = 16 - 18 \text{ MeV} \))

Liquid - Gas Transition at

**PHASE DIAGRAM of NUCLEAR MATTER**

- In-medium
  - **chiral effective field theory**
  - (3-loop in the free energy density)


- Pion-nucleon dynamics
- incl. delta isobars
- Short-distance NN contact terms
- Three-body forces

**PHASE DIAGRAM**

- Gas
- Liquid
- Critical point

**Phase Diagram Parameters**

- Temperature \( T \) [MeV]
- Baryon chemical potential \( \mu_B \) [MeV]
- Density \( \rho \) [fm\(^{-3}\)]

**Critical Points**

- Gas
- Liquid
- Gas - Liquid

**Chemical Potential and Temperature**

- \( \mu_B = 0 \)
- \( T = 16 \) MeV

**Symmetric Nuclear Matter**

- \( N = Z \)
PHASE DIAGRAM of NUCLEAR MATTER

Trajectory of CRITICAL POINT for asymmetric matter

... determined almost entirely by isospin dependent pion exchange dynamics

4
... from **QCD**
via
**CHIRAL EFFECTIVE FIELD THEORY** ...

... to the **NUCLEAR CHART**?
NUCLEAR MANY-BODY CALCULATIONS

... using NN and NNN interactions from Chiral Effective Field Theory

No-Core-Shell-Model results for $^{10}\text{B}$, $^{11}\text{B}$, $^{12}\text{C}$ and $^{13}\text{C}$ @ $\text{N}^2\text{LO}$

Navratil et al., PRL 99 (2007) 042501

systematic improvements with inclusion of 3-body interactions
DENSITY FUNCTIONAL STRATEGIES

... constrained by (chiral) symmetry breaking pattern of Low-Energy QCD

\[ E[\rho] = E_{\text{kin}} + \int d^3x \left[ \mathcal{E}^{(0)}(\rho) + \mathcal{E}_{\text{exc}}(\rho) \right] + E_{\text{coul}} \]

\[ \rho \rightarrow \rho(x) \]

\[ \text{Kohn - Sham equations} \]

\[ \mathcal{E}_{\text{exc}}(\rho) : \text{from in-medium Chiral Perturbation Theory ("Pionic fluctuations")} \]

\[ \mathcal{E}^{(0)}(\rho) : \text{Hartree mean field(s) from contact terms} \]

(equiv. to) \[ \text{strong SCALAR and VECTOR mean fields} \]

\[ \text{leading order IN-MEDIUM changes of QCD CONDENSATES} \]
Examples (part I)

- Strategy:
  - Calculate physics at long and intermediate distances using nuclear chiral effective field theory
  - Fix short distance constants (contact interactions) e.g. in Pb region
  - Predict systematics for all other nuclei

deviations (in %) between calculated and measured binding energies per nucleon ...

... and charge radii

- $\delta E/A$ (%)
- $\delta \langle r^2 \rangle^{1/2}$ (%)
Examples (part II)

charge density of $^{48}$Ca

Examples (part III):

DEFORMED NUCLEI

deviations (in %) between calculated and measured binding energies

Ground state deformations

Systematics through isotopic chains governed by isospin dependent forces from chiral pion dynamics

• **Gamow-Teller beta decays**

  interesting case: 
  
  $^{14}\text{C}$ \(\beta^-\) $^{14}\text{N}$ anomalously long lifetime (5739 y) enables radiocarbon dating

  Theoretically **not** understood on the basis of **two-nucleon** interactions only

  Solution: **chiral effective interaction** including **three-body force**


• **Spin-orbit interactions**

  Role of **2nd order tensor force** from **pion exchange** and **three-body interactions**


• **In-medium Chiral SU(3) dynamics** and **hypernuclei**

  **Weak \(\Lambda\)-nuclear spin-orbit coupling**

Anomally long beta decay lifetime of $^{14}$C


- Known lifetime of 5730 years enables radiocarbon dating
- But theoretical description using realistic nucleon-nucleon interactions overestimates the GT strength

Idea: Derive a density-dependent two-nucleon force from the leading-order chiral three-nucleon force

\[
T_{1/2} = \frac{1}{f(Z, E_0)} \frac{2\pi^3 \hbar^7 \ln 2}{m_e c^4 G_v^2} \frac{1}{g_A^*} |M_{GT}|^2
\]

Expt: $B(GT) \simeq |M_{GT}|^2 \simeq 10^{-6}$

- Large suppression of GT strength at $\rho_0$ due to chiral 3NF!

5 CHIRAL CONDENSATE at finite DENSITY

\( T = 0 \)

- Hellmann - Feynman theorem:  
  \[ \langle \Psi | \bar{q} q | \Psi \rangle = \langle \Psi | \frac{\partial \mathcal{H}_{\text{QCD}}}{\partial m_q} | \Psi \rangle = \frac{\partial \mathcal{E}(m_q; \rho)}{\partial m_q} \]

\[ \frac{\langle \bar{q} q \rangle_\rho}{\langle \bar{q} q \rangle_0} = 1 - \frac{\rho}{f_\pi^2} \left[ \frac{\sigma_N}{m_\pi^2} \left( 1 - \frac{3 p_F^2}{10 M_N^2} + \ldots \right) + \frac{\partial}{\partial m_\pi^2} \left( \frac{E_{\text{int}}(p_F)}{A} \right) \right] \]

- Sigma term: \( m_q \frac{\partial M_N}{\partial m_q} \)
- In-medium chiral effective field theory

(free) Fermi gas of nucleons

Nuclear interactions (dependence on pion mass)
CHIRAL CONDENSATE: DENSITY DEPENDENCE
Symmetric Nuclear Matter

- In-medium Chiral Effective Field Theory (NLO 3-loop)

Constrained by realistic nuclear equation of state

N. Kaiser, Ph. de Homont, W.W.

- Substantial change of symmetry breaking scenario
  between chiral limit \( m_q = 0 \) and physical quark mass \( m_q \sim 5 \text{ MeV} \)

- Nuclear Physics would be very different in the chiral limit!
**GOLDSTONE Bosons in Matter**

Chiral Symmetry:

\[ U_{\text{strong}}(\pi^\pm A) = \pm \frac{\rho_p - \rho_n}{4 f^2_\pi} + \ldots \]

\[ f_\pi \rightarrow f^*_\pi(\rho) \]

Deeply Bound States of Pionic Atoms

\[ f^*_\pi(\rho_0) \approx 0.8 f_\pi \sim 1 - \frac{\sigma_N}{2 m^2_\pi f^2_\pi} \rho_0 \]

Fingerprints of Chiral Symmetry Restoration

GSI

**exp.**: K. Suzuki et al. (2004)


PSI

**E. Friedman et al. (2004)**

Low Energy Pion-Nucleus Scattering

**exp.**:

- E. Friedman et al. (2004)

**theory**:


**deduced from exp.**

**theory pred.**

\( \sigma_N \approx 50 \text{ MeV} \)
Summary and Conclusions

- **Interface of Low-Energy QCD and Nuclear Physics:** Nuclear Chiral (Thermo-) Dynamics really works!

- Importance of **Two-Pion Exchange** processes in combination with Pauli principle

- **No** $\sigma$ and $\rho$ boson exchanges required

- **Three-Nucleon Forces** are natural part of nuclear chiral dynamics → **density dependent** effective two-body interactions

- Magnitude of **Chiral Condensate** drops approximately linearly up to normal nuclear matter density but tends to **saturate** for $\rho_0 < \rho < 2\rho_0$

**thanks to:**
- Paolo Finelli
- Salvatore Fiorilla
- Jeremy Holt
- Norbert Kaiser