Spin-Orbit Interactions in **Nuclei** and **Hypernuclei**

Wolfram Weise
Technische Universität München

- Phenomenology

- Aspects of Chiral Dynamics and Spin-Orbit Forces

N. Kaiser, Phys. Rev. C70 (2004) 034307
S. Fritsch, N. Kaiser, W.W., Nucl. Phys. A750 (2005) 259
N. Kaiser, W.W., Nucl. Phys. A804 (2008) 60
N. Kaiser, W.W., arXiv:0912.3207 [nucl-th]

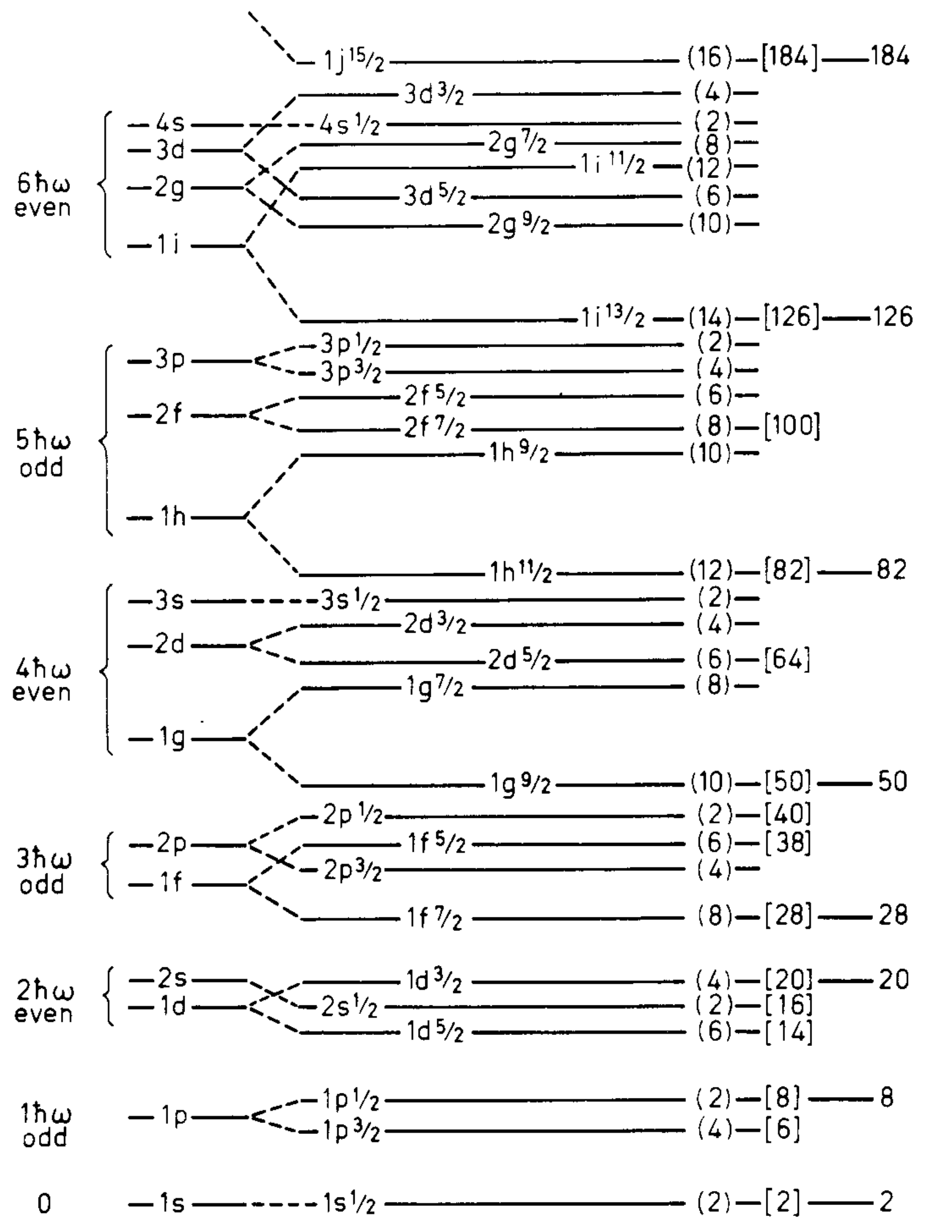
- Nuclei vs. Λ-Hypernuclei: understanding the Spin-Orbit Puzzle

N. Kaiser, W.W., Phys. Rev. C71 (2005) 015203
P. Finelli, N. Kaiser, D.Vretenar and W.W., Phys. Lett. B658 (2007) 90
N. Kaiser, W.W., Nucl. Phys. A804 (2008) 60
P. Finelli, N. Kaiser, D.Vretenar and W.W., Nucl. Phys. A831 (2009) 163

1.

Nuclear and Hypernuclear Spin-Orbit Forces: Phenomenology

Nuclear Shell Model Phenomenology



M. Goeppert-Mayer, J.H.D. Jensen (1955)

- **Spin-Orbit Interaction**

$$\Delta\mathcal{H}_{LS} = \frac{U_{LS}}{r}\frac{df(r)}{dr}\vec{L}\cdot\vec{s}$$

$$\vec{L} = -i\,\vec{r}\times\vec{\nabla} \qquad \vec{s} = \frac{1}{2}\vec{\sigma}$$

$$f(r) = \frac{\rho(r)}{\rho_0} = \left(1+\exp\frac{r-R}{a}\right)^{-1}$$

$$U_{LS} \simeq 30\,\mathrm{MeV}\cdot\mathrm{fm}^2$$

- **unusually large**:

one order of magnitude larger than expectation from Thomas term based on single particle potential and opposite sign

Skyrme Phenomenology

- **Energy Density** of slightly inhomogeneous nuclear matter

- **Spin-Orbit** part of energy density functional

$$\mathbf{\Delta}\mathcal{E}_{\mathbf{LS}}[\rho] = \mathbf{F}_{\mathbf{LS}}(\rho)\,\vec{\nabla}\rho \cdot \sum_{\alpha \in \mathbf{F}} \mathbf{\Psi}^{\dagger}_{\alpha}(\vec{\mathbf{r}})\,\mathbf{i}\vec{\sigma} \times \vec{\nabla}\mathbf{\Psi}_{\alpha}(\vec{\mathbf{r}})$$

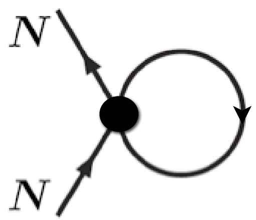
$$\mathbf{F}_{\mathbf{LS}}(\rho_{\mathbf{0}}) \simeq \mathbf{90\,MeV} \cdot \mathbf{fm^5}$$

- Relation between shell model phenomenology and Skyrme parametrization

$$\mathbf{2}\rho\,\mathbf{F}_{\mathbf{LS}}(\rho) = \mathbf{U}_{\mathbf{LS}}(\rho)$$

Phenomenology (part I): Strong **Scalar-Vector** Mean Fields

- **Short-distance** contribution to spin-orbit interaction



$$\mathbf{F}_{LS}^{sr} = \frac{\mathbf{G_S + G_V}}{\mathbf{4M_N^*}} = \frac{\mathbf{1}}{\mathbf{4M_N^*}}\left(\frac{\mathbf{g}_\sigma^2}{\mathbf{m}_\sigma^2} + \frac{\mathbf{g}_\omega^2}{\mathbf{m}_\omega^2}\right)$$

- **Equivalent** descriptions:
  - "**sigma**-plus-**omega**" boson exchange models á la Walecka
  - **contact** terms in NN Effective Field Theory
  - **strong scalar-vector** mean fields from QCD Sum Rules

$$\Sigma_S = -G_S\,\rho_S \qquad \Sigma_V = G_V\,\rho$$

$$\frac{\Sigma_S}{\Sigma_V} \simeq \frac{-\sigma_N}{4(m_u + m_d)}\left(\frac{\rho_S}{\rho}\right) \sim -1 \qquad G_S \simeq \frac{\sigma_N\, M_N}{m_\pi^2\, f_\pi^2} \simeq 10\,\mathrm{fm}^2$$

(Cohen, Furnstahl; PRL 67 (1991) 961)

- **Result**: empirical $\mathbf{F_{LS}}(\rho_0) \simeq \mathbf{90\,MeV \cdot fm^5}$
  "understood" at Hartree level in terms of **short-distance** NN dynamics

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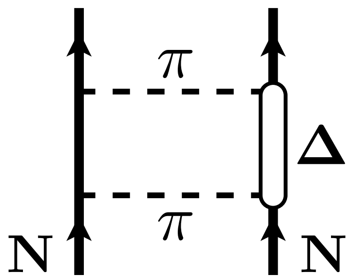
Phenomenology (part II): Fujita-Miyazawa Mechanism

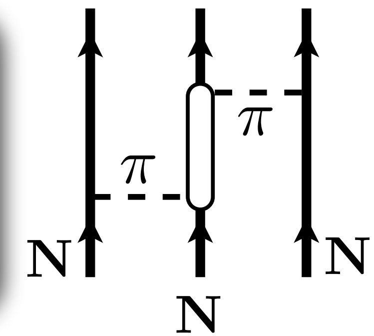
(J. Fujita, H. Miyazawa; Prog. Theor. Phys. 17 (1957) 360)



large spin-isospin polarizabilty

$$\beta_\Delta = \frac{g_A^2}{f_\pi^2 (M_\Delta - M_N)} \sim 5\,\mathrm{fm}^3$$

$\longleftrightarrow$ **strong 3-body interaction**



- **Pionic Van der Waals** - type intermediate range central potential

$$V_c(r) = -\frac{9\, g_A^2}{32\pi^2 f_\pi^2}\, \beta_\Delta\, \frac{e^{-2m_\pi r}}{r^6}\, P(m_\pi r)$$

N. Kaiser, S. Fritsch, W.W. Nucl. Phys. A750 (2005) 259

- **Large** contribution to **spin-orbit interaction**

$$F_{LS}^{(\Delta)} \simeq (50 - 70)\,\mathrm{MeV\,fm}^5$$

- How can there be a coexistence in the spin-orbit force between **short-distance** and Fujita-Miyazawa **two-pion exchange** mechanisms ?

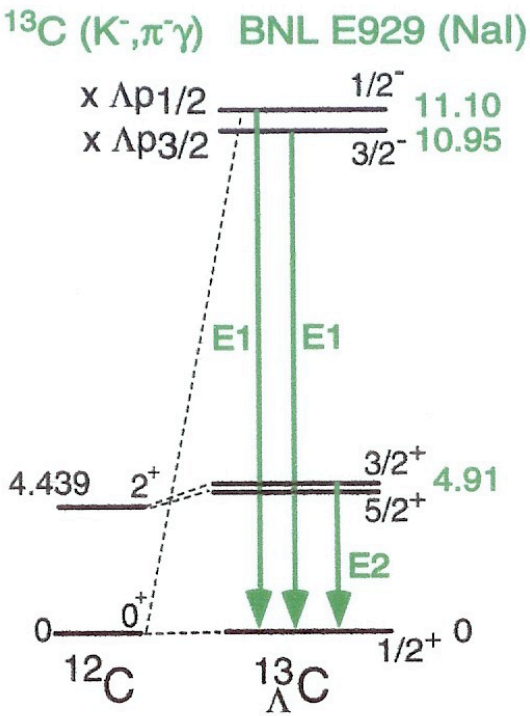
Spin-Orbit Coupling in Λ-Hypernuclei

- **Central Potential**

$$\mathbf{U}_0^{(\Lambda)} \simeq \frac{1}{2}\mathbf{U}_0^{(N)}$$

- **Spin-Orbit Potential**

$$\mathbf{U}_{LS}^{(\Lambda)} \leq \frac{1}{20}\mathbf{U}_{LS}^{(N)}$$



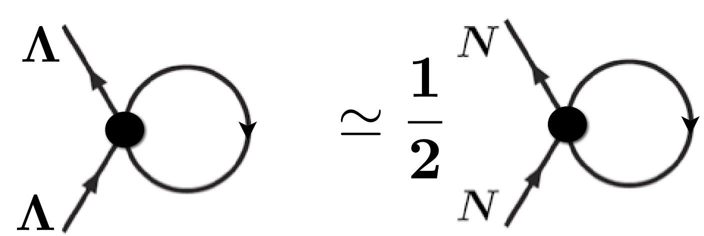
PRL 86 (2001) 4255

$$\mathbf{E}_\Lambda(\mathbf{p}_{1/2}) - \mathbf{E}_\Lambda(\mathbf{p}_{3/2}) = (\mathbf{152 \pm 54 \pm 36})\,\mathbf{keV}$$

unusually small:
more than 20 times smaller
than the $\mathbf{p}_{1/2} - \mathbf{p}_{3/2}$
spin-orbit splitting (~ 5 MeV)
for nucleons in nuclei

- ... cannot be understood in terms of short-distance dynamics

scalar-vector mean fields



2.
Chiral Dynamics
and
Spin-Orbit Interactions

CHIRAL DYNAMICS APPROACH to the NUCLEAR MANY-BODY PROBLEM

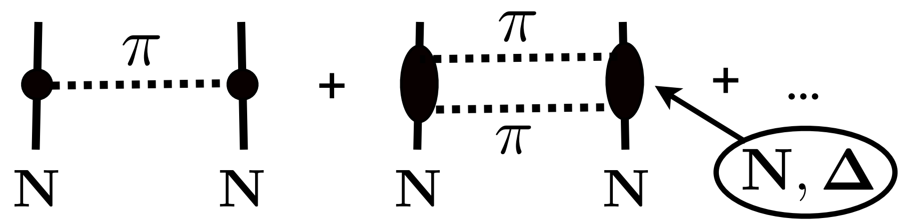
- **Relevant "small" scales**:

$$p_F \sim 2\,m_\pi \sim M_\Delta - M_N << 4\pi\, f_\pi \sim 1\,\text{GeV}$$

- **PIONS** (and **DELTA** isobars) as **explicit** degrees of freedom

- **IN-MEDIUM CHIRAL PERTURBATION THEORY**

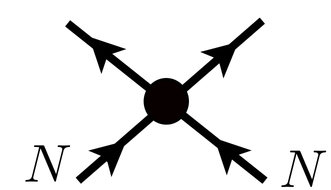
➡ pion exchange in presence of filled Fermi sea



" in medium"

2nd order TENSOR force + nucleon's SPIN-ISOSPIN polarizability

➡ short-distance dynamics: contact interactions

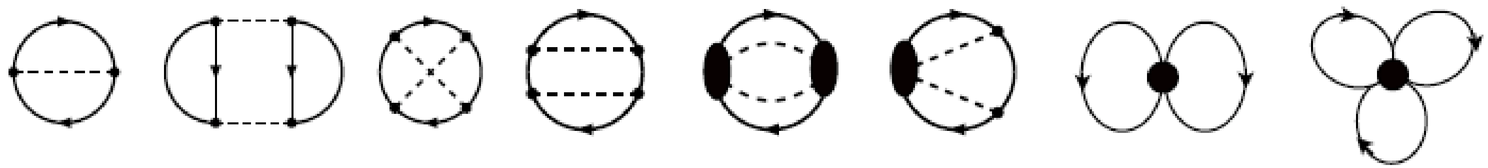


CHIRAL DYNAMICS APPROACH to the **NUCLEAR MANY-BODY PROBLEM** (contd.)

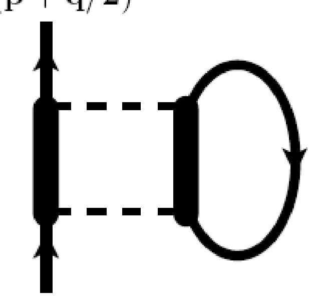
- Compute **energy density** $\mathcal{E}(p_F)$ using

  in-medium chiral perturbation theory
  (3-loop order)

N. Kaiser, S. Fritsch, W.W. (2002-2005)



- result: **realistic nuclear matter** equation of state
- **spin-dependent nucleon self-energy**
  in (slightly) inhomogeneous nuclear matter
- 2nd order **tensor force** from iterated one-pion exchange



produces strong **spin-orbit term**

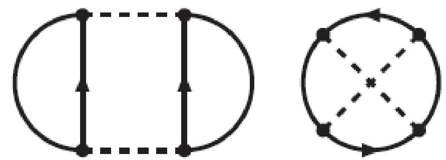
$$\Sigma_{LS}(\vec{p},\vec{q}) = U_{LS}(p_F)\, i\,\vec{s}\cdot(\vec{q}\times\vec{p})$$

... of "wrong" sign!

Iterated ONE-PION EXCHANGE

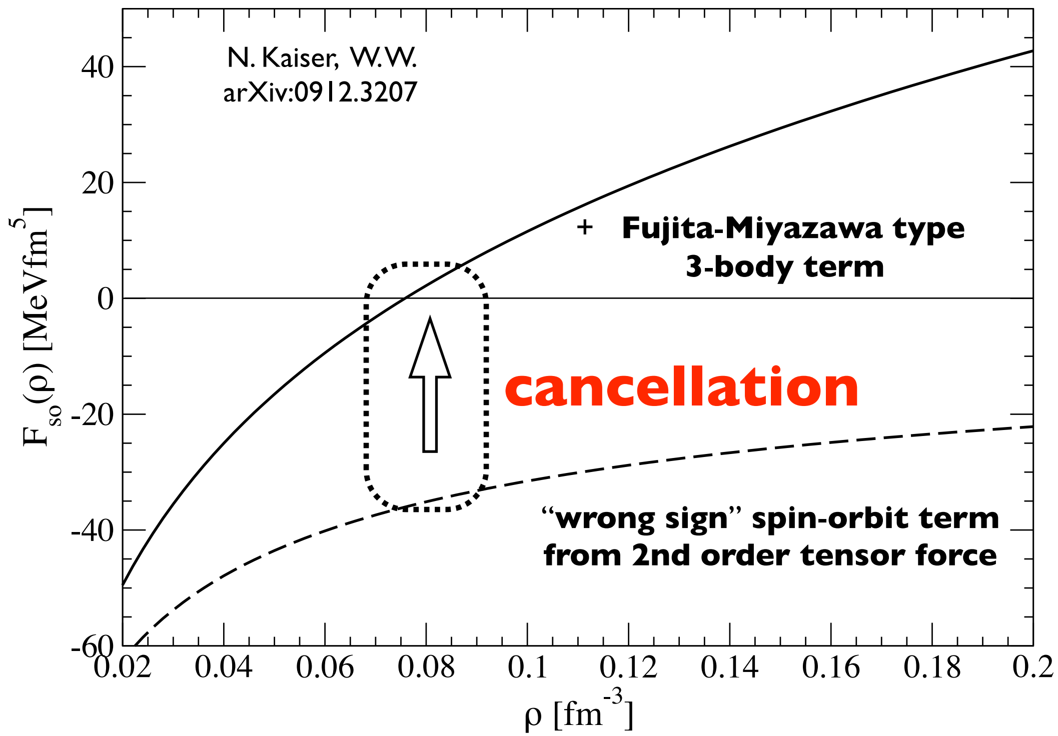
- Kaiser´s “WRONG SIGN” Spin-Orbit Interaction -

N. Kaiser
PRC70 (2004) 034307

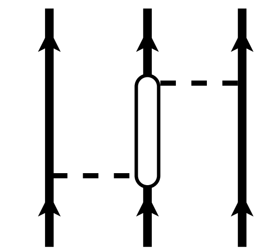


N. Kaiser, W.W.
NPA804 (2008) 60

$$\mathbf{F}_{\mathbf{LS}}^{(\pi\pi)}(\rho) = -\frac{3}{64\pi}\frac{\mathbf{M_N}}{\mathbf{m}_\pi}\left(\frac{\mathbf{g_A}}{\mathbf{f}_\pi}\right)^4\left[\frac{5}{16}\frac{\mathbf{m}_\pi^4}{\mathbf{p_F^4}}\ln\left(1+\frac{4\mathbf{p_F^2}}{\mathbf{m}_\pi^2}\right)-\frac{\mathbf{m}_\pi^3}{\mathbf{p_F^3}}\arctan\frac{2\mathbf{p_F}}{\mathbf{m}_\pi}+\frac{3}{4}\frac{\mathbf{m}_\pi^2}{\mathbf{p_F^2}}\right]+\mathbf{exch.}$$



...+



$$\mathbf{F}_{\mathbf{LS}}^{(\boldsymbol{\Delta})}(\rho) = \frac{\mathbf{m}_\pi}{64\pi^2(\mathbf{M}_{\boldsymbol{\Delta}}-\mathbf{M_N})}\left(\frac{\mathbf{g_A}}{\mathbf{f}_\pi}\right)^4\mathcal{F}\left(\frac{\mathbf{p_F}}{\mathbf{m}_\pi}\right)$$

- Strong cancellation between Miyazawa type **two-pion exchange / three-body terms** and **2nd order tensor** contributions

Intermediate Summary: Balance of Spin-Orbit Terms

- **Three** major contributions to **nuclear** spin-orbit interactions:

  - **Short-distance** component (strong **scalar** and **vector** fields) ← **survives**
  - "**Wrong-sign**" spin-orbit term generated by 2nd order **pion exchange tensor force**
  - **Three-body** term induced by **two-pion exchange** with intermediate **Delta**(1230) (á la Fujita-Miyazawa)

  ↕ **cancellation** (between the second and third terms)

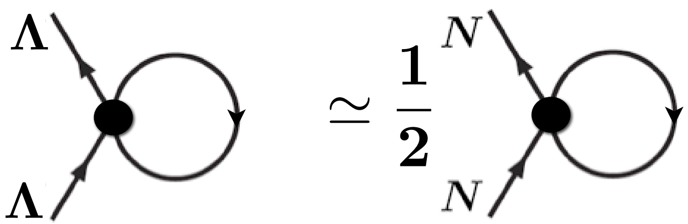
- Learn more about underlying mechanisms by comparison with **hypernuclear** spin-orbit forces (different balance of terms)

3.

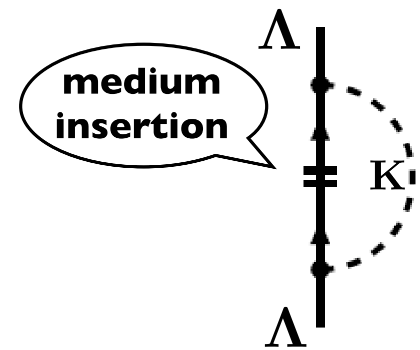
Chiral SU(3) Dynamics
and
Spin-Orbit Interaction
in
Λ - Hypernuclei

Λ HYPERON SELF-ENERGY

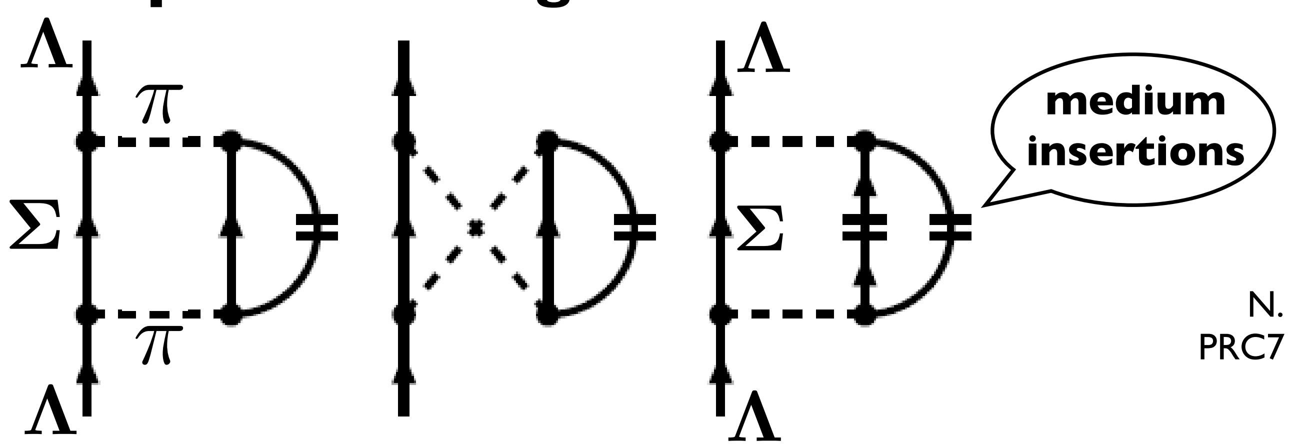
- **short-distance** (contact) terms
  (e.g. **scalar-vector mean fields**)



- **small**:
  (K exchange **Fock term**)



- **large**: **two-pion exchange** mechanisms



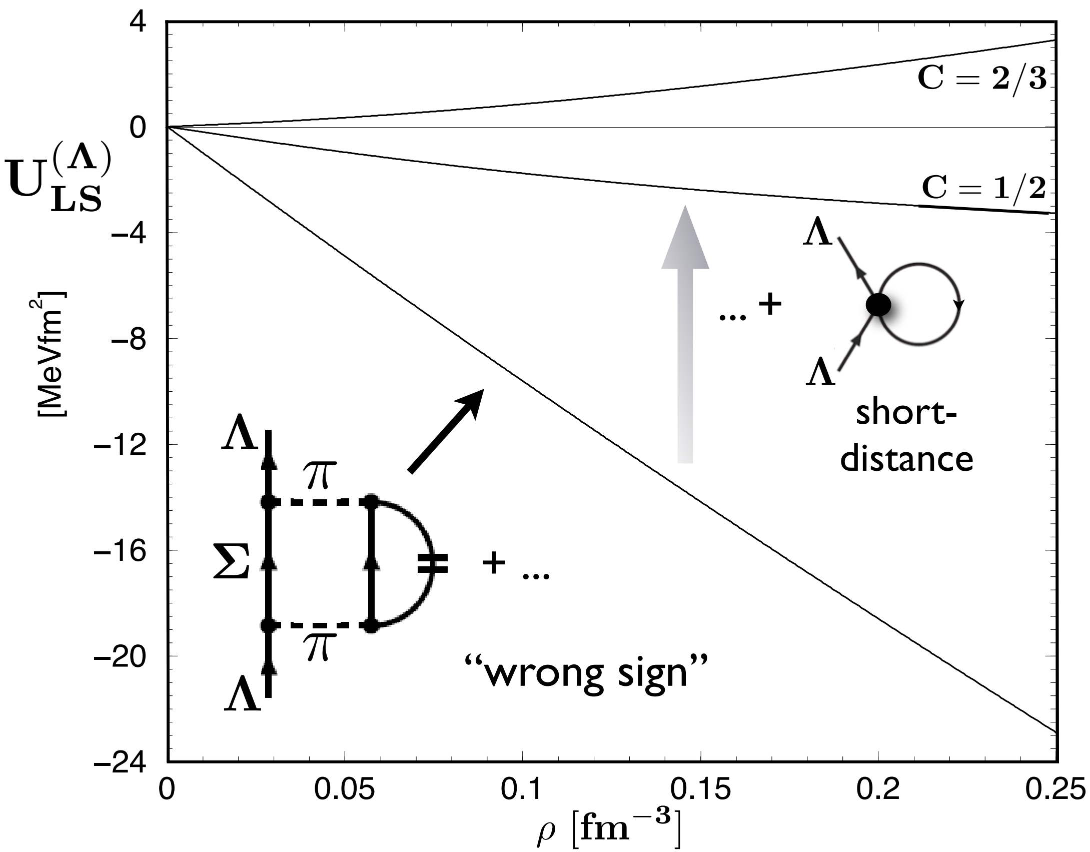
N. Kaiser, W.W.
PRC71 (2005) 015203

- 2nd order **tensor force** produces **spin-orbit term**

$$\Sigma^{(\Lambda)}_{\mathbf{LS}}(\vec{\mathbf{p}},\vec{\mathbf{q}}) = \mathbf{U}^{(\Lambda)}_{\mathbf{LS}}(\mathbf{p_F})\, \mathbf{i}\, \vec{\mathbf{s}} \cdot (\vec{\mathbf{q}} \times \vec{\mathbf{p}})$$

Spin-Orbit Coupling in Λ-Hypernuclei

- **cancellation** between
  short-distance (contact) and **two-pion exchange** terms



$$U_{LS}^{(\Lambda)} = C \left(\frac{M_N}{M_\Lambda}\right)^2 U_{LS}^{(N)}$$

for **short-distance** terms

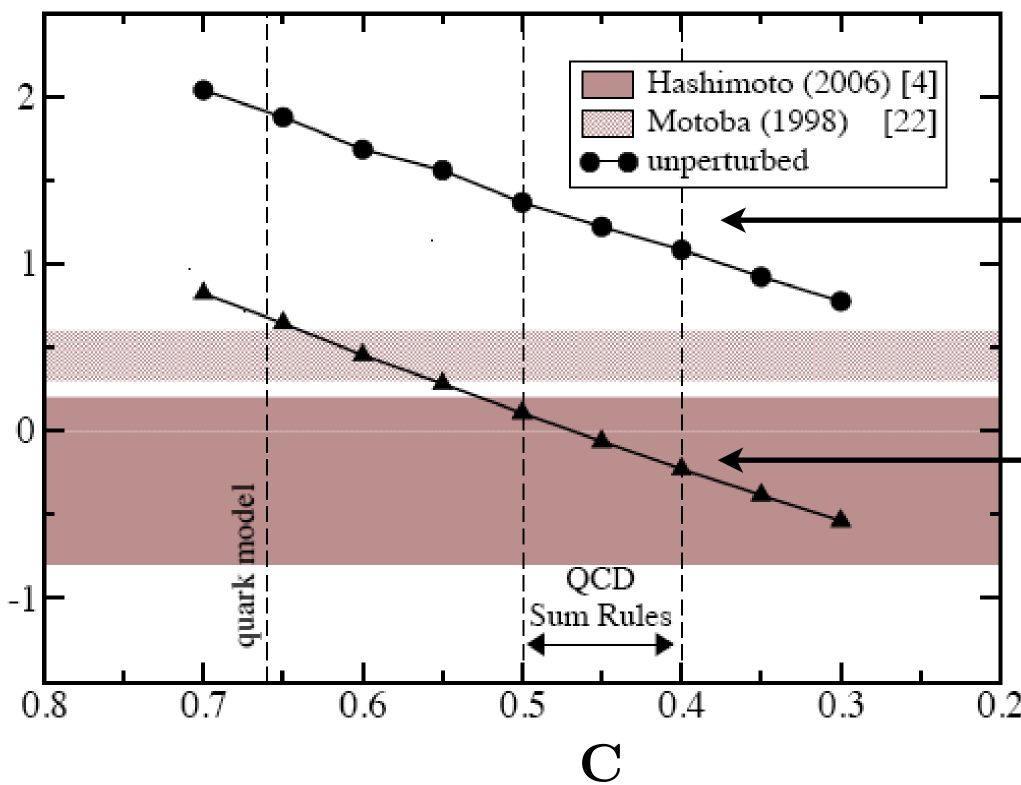
N. Kaiser, W.W.
PRC71 (2005) 015203

- **NO three-body** terms á la Miyazawa: no hyperon Fermi sea

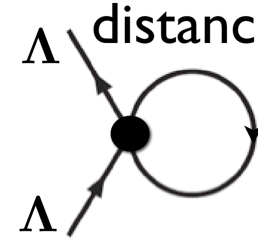
Finite Systems

- translate **in-medium chiral dynamics → density functional**
  add surface terms (calculable)
- very satisfactory results for systematics of **nuclei** and **hypernuclei**
- example: $^{16}\mathrm{O}_\Lambda$

$$\varepsilon_\Lambda(p_{1/2}) - \varepsilon_\Lambda(p_{3/2})$$
$$[\mathrm{MeV}]$$



short-distance

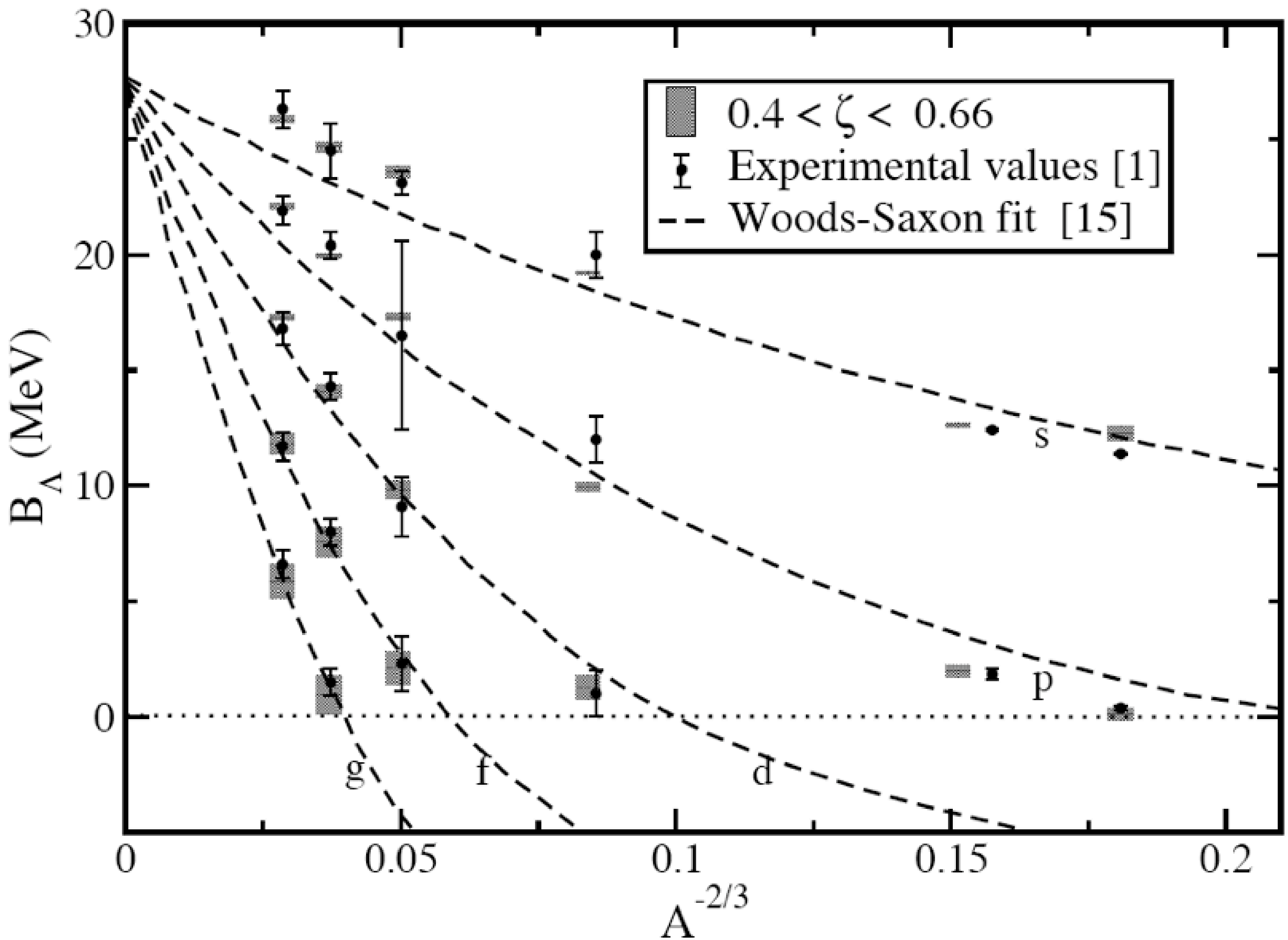


+ $\pi\pi$
2nd order
tensor

P. Finelli, N. Kaiser, D.Vretenar, W.W.
PLB658 (2007) 90

Finite Systems (contd.)

P. Finelli, N. Kaiser, D. Vretenar, W.W.
Nucl. Phys. A831 (2009) 163



(contd.)

Nucleus	$\epsilon_{s.p.}$	Expt.	FKVW	QMC	FHNC	SK	BHF	RMFI	RMFII
${}^{13}_{\Lambda}$C	$1s_{1/2}$	11.38 ± 0.05	12.3	–	8.3	11.7	13.7	12.5	11.7
	$1p_{3/2}$	0.38 ± 0.1	0.1	–	–	0.9	1.4	1.1	1.1
	$1p_{1/2}$		0.0					0.8	0.0
${}^{16}_{\Lambda}$O	$1s_{1/2}$	12.42 ± 0.05	12.6	16.2	12.00	13.3	15.5	12.9	12.8
	$1p_{3/2}$	1.85 ± 0.06	2.0	6.4	1.8	3.0	3.7	3.3	2.8
	$1p_{1/2}$		1.9	6.4				3.0	1.4
${}^{40}_{\Lambda}$Ca	$1s_{1/2}$	20.0 ± 1.0	18.9	20.6	20.0	18.0	20.7	19.0	17.6
	$1p_{3/2}$	12.0 ± 1.0	10.1	13.9	10.6	10.1	11.5	10.7	9.1
	$1p_{1/2}$		10.1	13.9				10.5	7.8
	$1d_{5/2}$	1.0 ± 1.0	1.6	5.5	1.6	1.6	2.0	2.7	1.5
	$1d_{3/2}$		0.9	5.5				2.4	1.5
${}^{89}_{\Lambda}$Y	$1s_{1/2}$	23.1 ± 0.5	23.4	24.0	23.3	21.1	24.1	23.7	23.2
	$1p_{3/2}$	16.5 ± 4.1	17.2	19.4	16.9	15.6	17.8	17.6	17.2
	$1p_{1/2}$		17.2	19.4				17.4	16.3
	$1d_{5/2}$	9.1 ± 1.3	10.2	13.4	10.1	9.1	10.4	10.7	10.3
	$1d_{3/2}$		9.8	13.4				10.5	8.9
	$1f_{7/2}$	2.3 ± 1.2	2.8	6.5	–	2.1	2.4	3.7	3.1
	$1f_{5/2}$		2.0	6.4				3.4	1.0

P. Finelli, N. Kaiser, D.Vretenar, W.W. Nucl. Phys. A831 (2009) 163

Binding energies (in MeV) of single-Λ levels in ${}^{13}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, ${}^{40}_{\Lambda}$Ca and ${}^{89}_{\Lambda}$Y. Experimental energies [1] are shown in comparison with the results of the present calculations, using the input parameters of Table 1 and $\zeta = 0.5$ (column FKVW). Also listed are results of five different models: Quark Meson Coupling (QMC) [12,13], Fermi Hypernetted Chain (FHNC) [18], Skyrme (SK) [16], Brueckner-Hartree-Fock (BHF) [19] with the Njimegen SC97F potential [50], and RMF models with a tensor coupling [11] (RMFI with $f^{\Lambda}_{\omega}/g^{\Lambda}_{\omega} = -1$) and density-dependent couplings [14] (RMFII).

(contd.)

P. Finelli, N. Kaiser, D. Vretenar, W.W.
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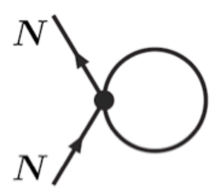
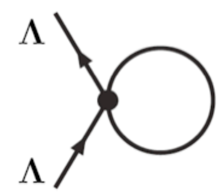
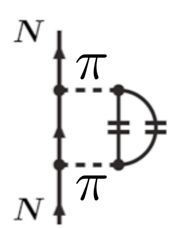
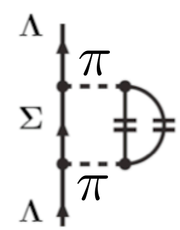
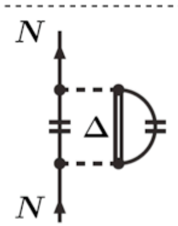
Nucleus	$\epsilon_{s.p.}$	Expt.	FKVW	QMC	FHNC	SK	BHF	RMFI	RMFII
$^{139}_{\Lambda}$La	$1s_{1/2}$	24.5 ± 1.2	24.7	–	–	22.1	25.3	25.2	25.2
	$1p_{3/2}$	20.4 ± 0.6	20.0	–	–	17.9	20.5	20.4	20.5
	$1p_{1/2}$		20.0					20.4	20.2
	$1d_{5/2}$	14.3 ± 0.6	14.3	–	–	12.8	14.5	14.8	14.9
	$1d_{3/2}$		14.1					14.6	14.1
	$1f_{7/2}$	8.0 ± 0.6	8.0	–	–	6.9	7.8	8.6	8.5
	$1f_{5/2}$		7.4					8.4	7.1
	$1g_{9/2}$	1.5 ± 0.6	1.5	–	–	0.6	0.6	2.4	2.2
	$1g_{7/2}$		0.5					2.0	0.2
$^{208}_{\Lambda}$Pb	$1s_{1/2}$	26.3 ± 0.8	25.8	26.9	27.6	23.1	26.5	26.5	27.2
	$1p_{3/2}$	21.9 ± 0.6	22.0	24.0	22.8	19.6	22.4	22.7	23.4
	$1p_{1/2}$		22.0	24.0				22.6	23.1
	$1d_{5/2}$	16.8 ± 0.7	17.4	20.1	17.4	15.4	17.5	18.0	18.5
	$1d_{3/2}$		17.3	20.1				17.9	17.9
	$1f_{7/2}$	11.7 ± 0.6	12.2	15.4	–	10.5	11.8	12.7	13.2
	$1f_{5/2}$		11.8	15.4				12.5	12.1
	$1g_{9/2}$	6.6 ± 0.6	6.5	10.1	–	5.1	5.6	7.1	7.5
	$1g_{7/2}$		5.8	10.1				6.9	5.8

SUMMARY

Hypernuclear vs. **Nuclear Spin-Orbit puzzle**

. . . **not** a puzzle in-medium Chiral Effective Field Theory

	Nucleons		Hyperons		
short-distance Hartree		**+**		**+**	QCD sum rules NN contact terms scalar-vector MF
in-medium ChPT (2nd order tensor)		**−**		**−**	"wrong sign" spin-orbit (Kaiser)
in-medium ChPT (3-body)		**+**	no contributions	**×**	Fujita- Miyazawa mechanism
	large and positive		small		

- Comparisons between **nuclei** and **hypernuclei** useful in studying the role of 3-body interaction mechanisms

Appendix: Short Note on **Central Repulsion in the Sigma-Nuclear Potential**

PHYSICAL REVIEW C **71**, 068201 (2005)

Chiral dynamics of Σ hyperons in the nuclear medium

N. Kaiser
Physik-Department T39, Technische Universität München, D-85747 Garching, Germany
(Received 3 February 2005; published 6 June 2005)

Using SU(3) chiral perturbation theory, we calculate the density-dependent complex mean field $U_\Sigma(k_f) + i\,W_\Sigma(k_f)$ of a Σ hyperon in isospin-symmetric nuclear matter. The leading long-range ΣN interaction arises from one-kaon exchange and from two-pion exchange with a Σ or a Λ hyperon in the intermediate state. We find from the $\Sigma N \to \Lambda N$ conversion process at nuclear matter saturation density $\rho_0 = 0.16\,\mathrm{fm}^{-3}$ an imaginary single-particle potential of $W_\Sigma(k_{f0}) = -21.5$ MeV, in fair agreement with existing empirical determinations. The genuine long-range contributions from iterated (second order) one-pion exchange with an intermediate Σ or Λ hyperon sum to a moderately repulsive real single-particle potential of $U_\Sigma(k_{f0}) = 59$ MeV. Recently measured (π^-, K^+) inclusive spectra related to Σ^- formation in heavy nuclei give evidence for a Σ-nucleus repulsion of similar size. Our results suggest that the net effect of the short-range ΣN interaction on the Σ nuclear mean field could be small.

Basic Elements of Σ Self-Energy in Nuclear Matter

from SU(3) in-medium chiral effective field theory

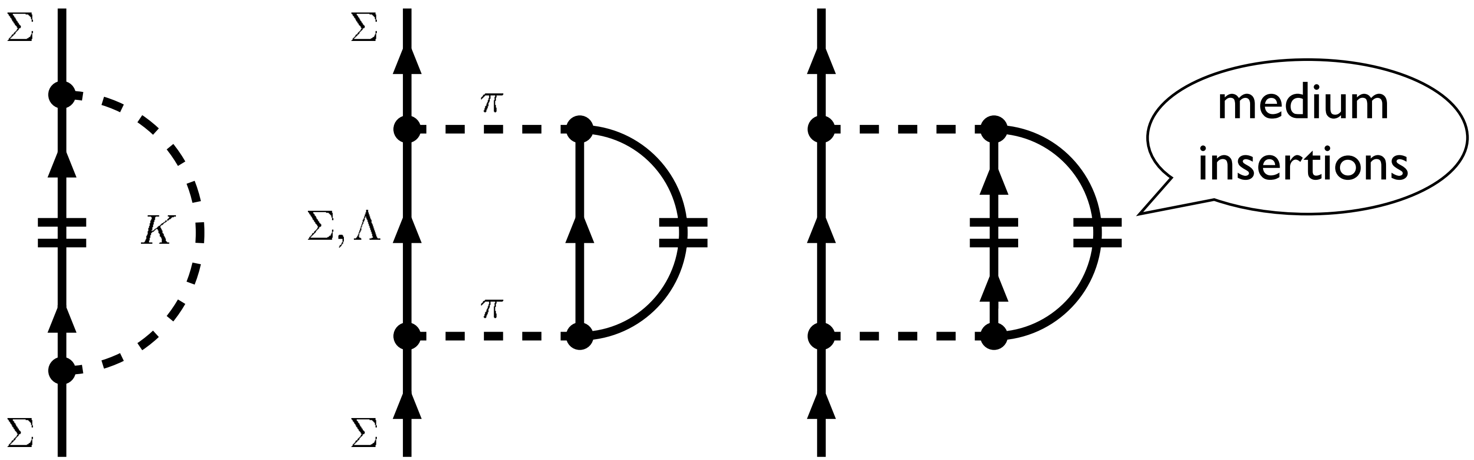
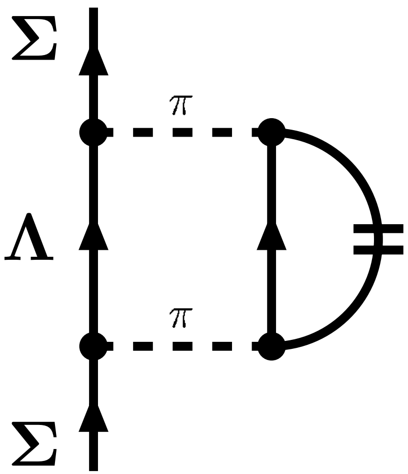


FIG. 1. One-kaon exchange Fock diagram and iterated one-pion exchange Hartree diagrams with Σ or Λ hyperons in the intermediate state. The horizontal double line symbolizes the filled Fermi sea of nucleons, i.e., the medium insertion $-\theta(k_f - |\vec{p}\,|)$ in the in-medium nucleon propagator [15]. Effectively, the medium insertion sums up hole propagation and the absence of particle propagation below the Fermi surface $|\vec{p}\,| < k_f$.

Complex Σ-Nuclear Potential

from iterated pion exchange (2nd order tensor force)



$$\mathbf{M_\Sigma - M_\Lambda = 77.5\,MeV}$$

introduce average baryon mass

$$\mathbf{M_B = \frac{1}{4}(2M_N + M_\Lambda + M_\Sigma) = 1047\,MeV}$$

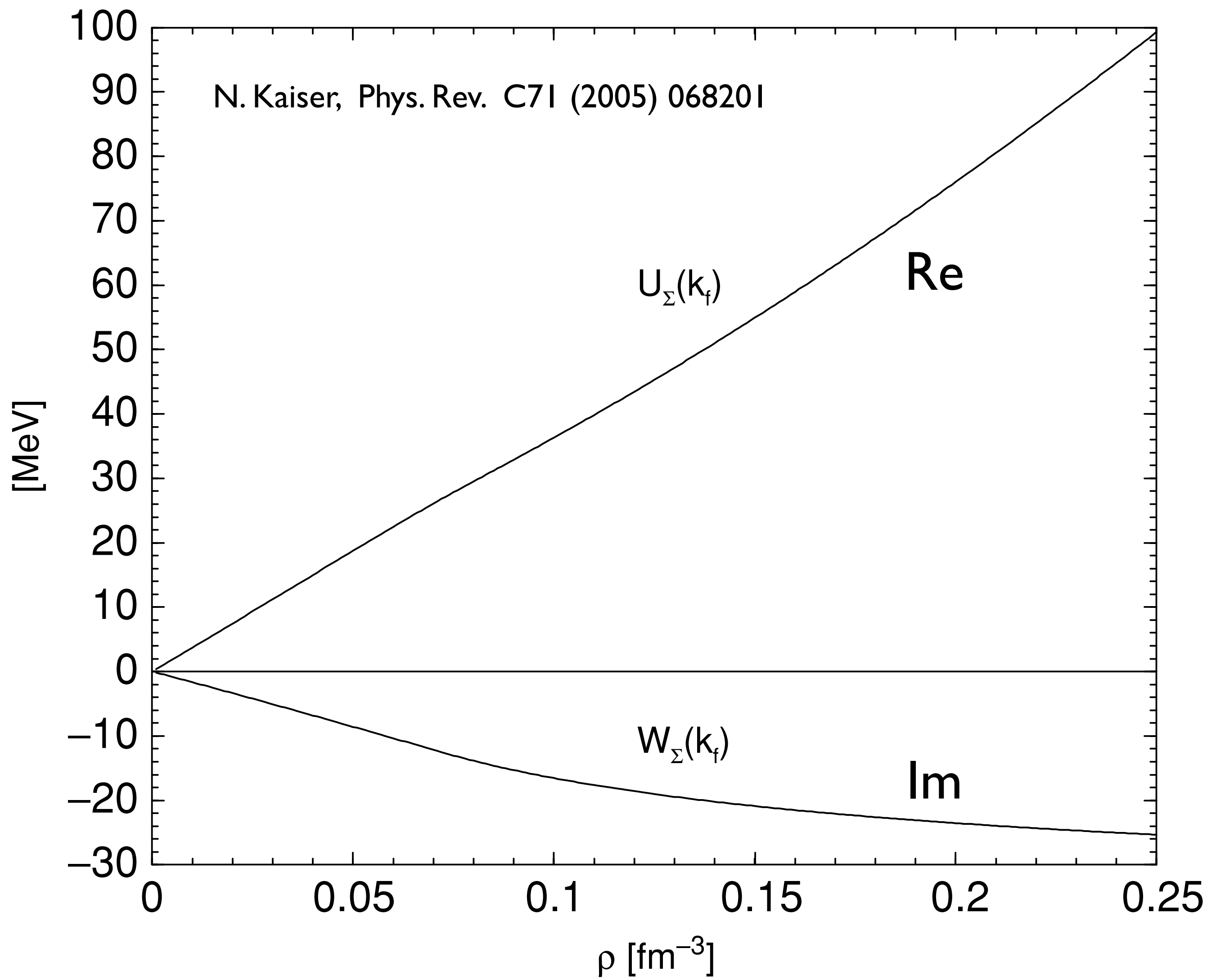
and $\mathbf{M_\Sigma - M_\Lambda = \Delta^2/M_B}$

perform principal-value integration

$$U_\Sigma(k_f)^{(2\pi\Lambda)} + i\,W_\Sigma(k_f)^{(2\pi\Lambda)} = \frac{D^2 g_A^2 M_B m_\pi^4}{48\pi^3 f_\pi^4}\,\Psi\left(\frac{k_f^2}{m_\pi^2}, \frac{\Delta^2}{m_\pi^2}\right)$$

$$\Psi(u,\delta) = -(\delta+3)\sqrt{u} - \frac{i}{4}(u+2\delta+6)\sqrt{u(4\delta+u)}$$
$$+ (2u+\delta^2+4\delta+3)\left\{\arctan\frac{\sqrt{u}}{1+\delta} + i\ln\frac{2+2\delta+u+\sqrt{u(4\delta+u)}}{2[(1+\delta)^2+u]^{1/2}}\right\}$$

Real and **imaginary** part of Σ **-Nuclear Potential**



- Hartree type contributions from short-distance physics (contact terms) estimated to be small (⟶ QCD Sum Rules)