QCD SUM RULES for VECTOR MESONS, revisited

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with

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... including an update on:
In-Medium QCD Sum Rules and BR Scaling

I.

Introductory Notes

about

Chiral Order Parameters

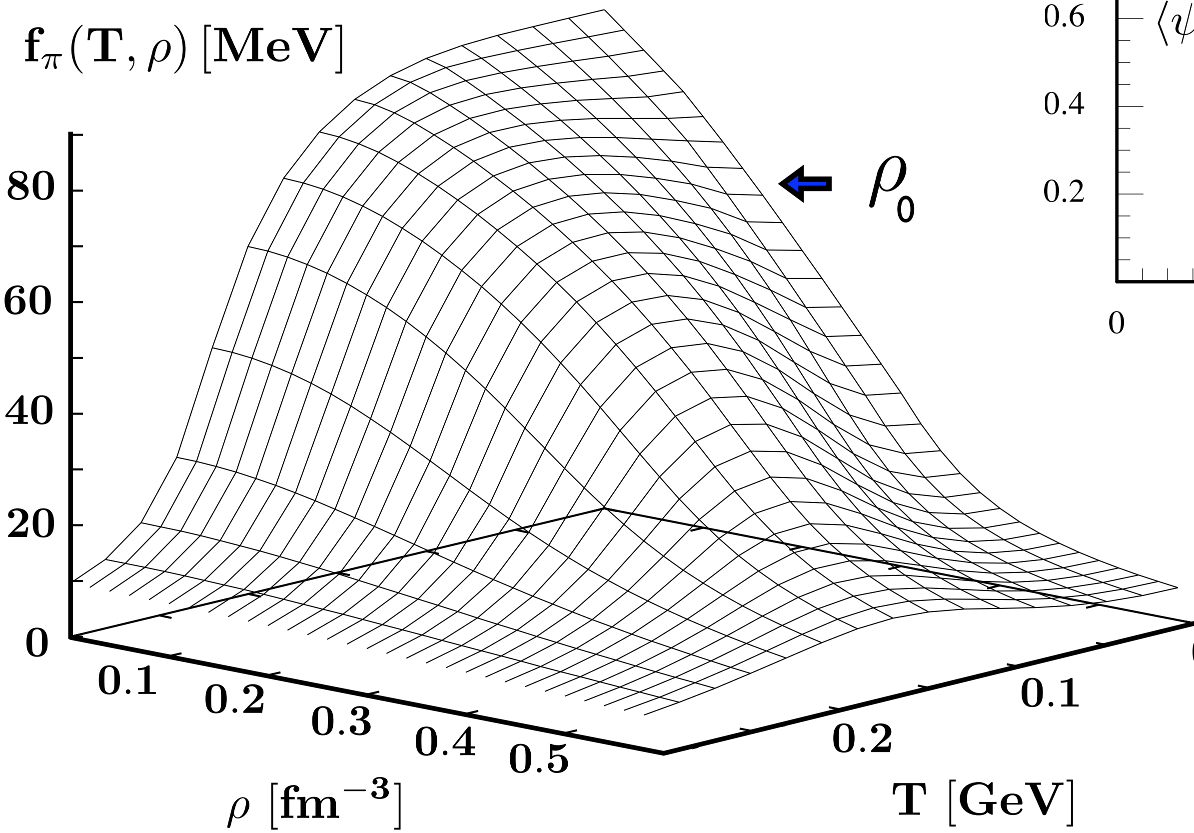
and

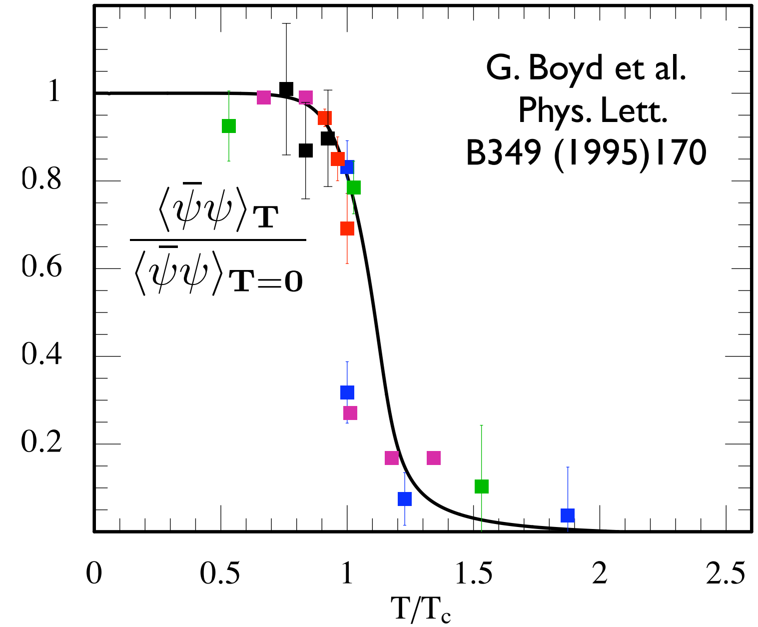
Current Correlation Functions

1.1 CHIRAL ORDER PARAMETER

- **Pion decay constant** and **Chiral Condensate**

dependence on
temperature and baryon density
$(\rho_0 \simeq 0.16\,\mathbf{fm^{-3}})$





S. Klimt, M. Lutz, W.W.
Phys. Lett. B249 (1990) 386

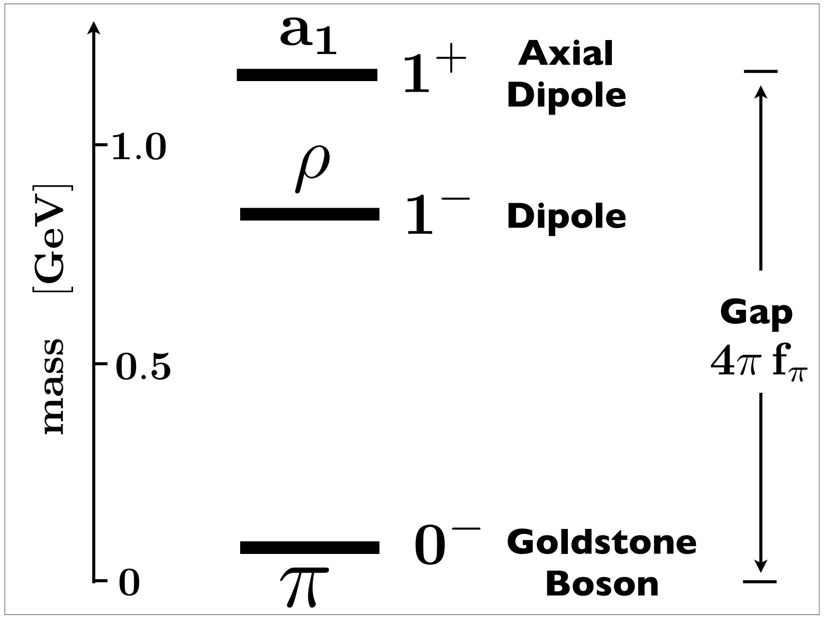
C. Ratti, M. Thaler, W.W.
Phys. Rev. D73 (2006) 014019

$$\frac{\mathbf{f_\pi^2(T,\rho)}}{\mathbf{f_\pi^2(0)}} \sim \frac{\langle\bar{\mathbf{q}}\mathbf{q}\rangle_{\mathbf{T},\rho}}{\langle\bar{\mathbf{q}}\mathbf{q}\rangle_0} = \mathbf{1} - \frac{\mathbf{T^2}}{\mathbf{8\,f_\pi^2}} - \frac{\sigma_\mathbf{N}}{\mathbf{m_\pi^2\,f_\pi^2}}\,\rho + \;\ldots$$

nucleon "sigma" term

$$\sigma_\mathbf{N} \simeq \mathbf{45\,MeV}$$

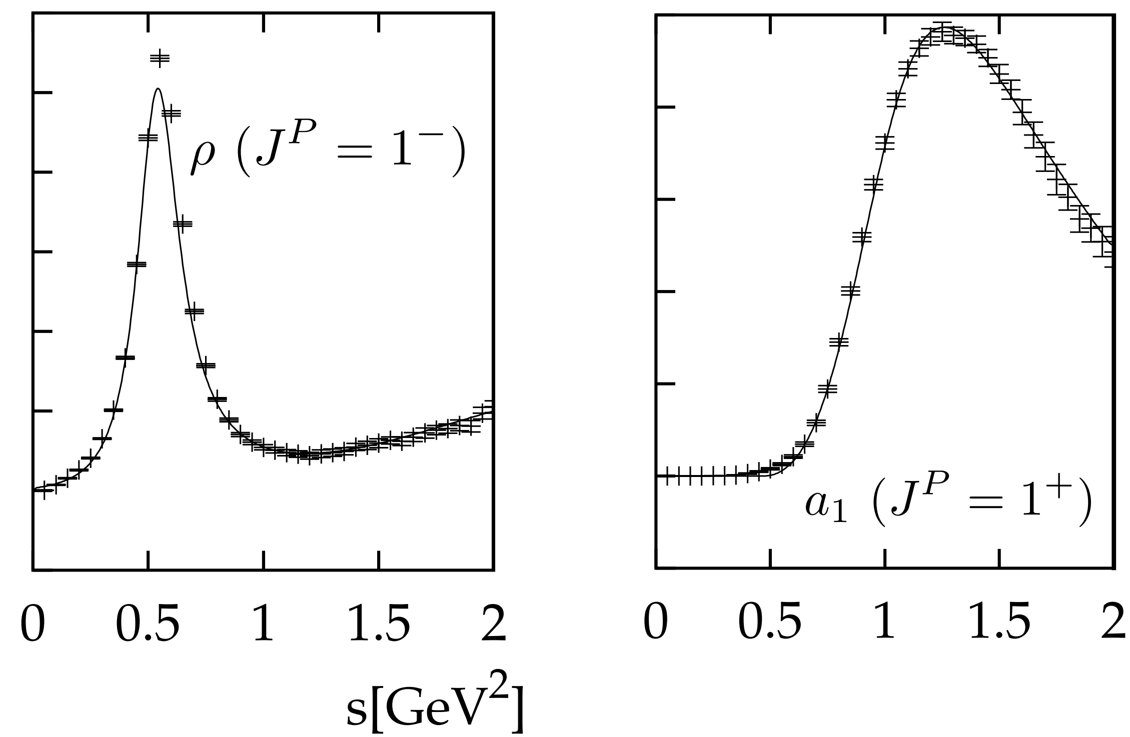
1.2 Spontaneous CHIRAL SYMMETRY Breaking and Current Algebra Relations



$$\mathbf{m_{a_1} = 1230 \pm 40\ MeV}$$
$$\mathbf{\Gamma_{a_1} \sim 400\ MeV}$$

$$\mathbf{m_\rho = 775.5 \pm 0.4\ MeV}$$
$$\mathbf{\Gamma_\rho \simeq 150\ MeV}$$

$$\frac{\mathbf{m_{a_1}}}{\mathbf{m_\rho}} \simeq \mathbf{1.6}$$



- Current Algebra
  Weinberg Sum Rules

$$\mathbf{m_{a_1} = \sqrt{2}\, m_\rho = 4\pi\, f_\pi}$$

- KSFR Relation

$$\mathbf{m_\rho^2 = 2\, g^2\, f_\pi^2} \qquad (\mathbf{g} = 2\pi)$$

1.3 **Vector** and **Axial Vector** CURRENT CORRELATION FUNCTIONS

$$\Pi^V_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|\mathcal{T}[\mathbf{V}_\mu(x)\mathbf{V}_\nu(0)]|0\rangle$$

$$\Pi^A_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|\mathcal{T}[\mathbf{A}_\mu(x)\mathbf{A}_\nu(0)]|0\rangle$$

$$\Pi_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi(q^2)$$

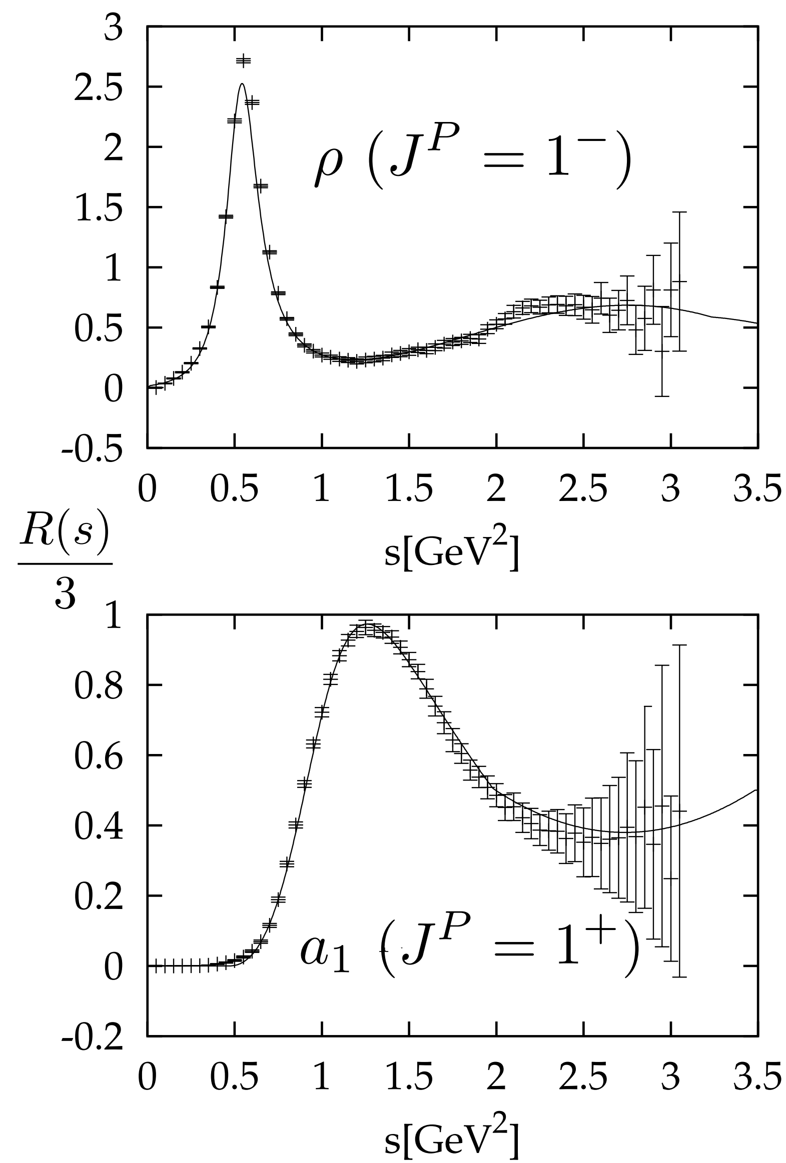
- Spectral Distributions:

$$R(s) = -\frac{12\pi}{s} Im\,\Pi(s = q^2)$$

$$= \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

M.Barate et al. (ALEPH): Eur. Phys. J. C4 (1998) 409;
K.Ackerstaff et al. (OPAL): Eur. Phys. J. C7 (1999) 571;

sum rule analysis:
E. Marco, W.W. : Phys. Lett. B 482 (2000) 87

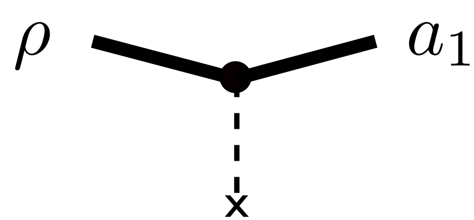


1.4 FINITE TEMPERATURE : VECTOR - AXIAL VECTOR MIXING

- Thermal Current Correlation Functions

$$\Pi^{\mu\nu}_{\mathbf{J}}(q;T) = \frac{i\sum_n \int d^4x\, e^{iq\cdot x}\langle n|\mathcal{T}[\mathbf{J}^\mu(x)\mathbf{J}^\nu(0)]\,e^{-H/T}|n\rangle}{\sum_n \langle n|e^{-H/T}|n\rangle}$$

- $\rho - a_1$ mixing:



pion
(from heat bath)

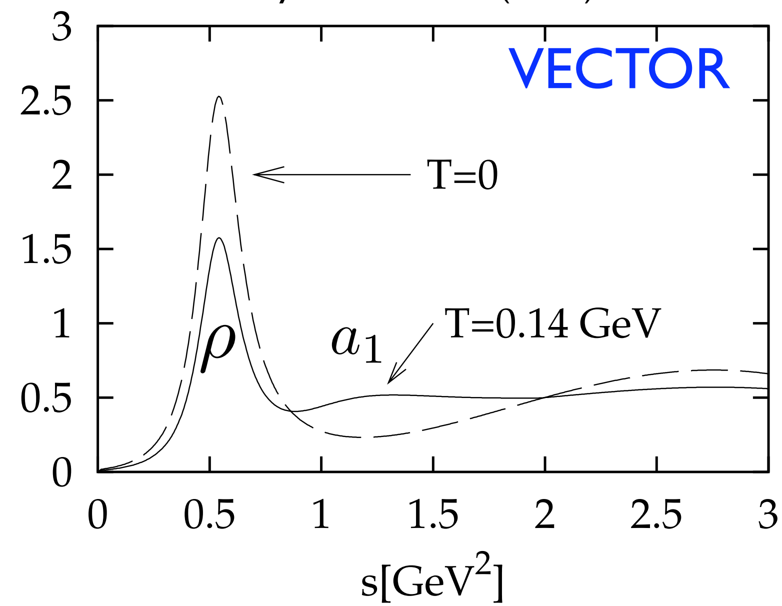
$$\Pi^{\mu\nu}_{\mathbf{V}}(q;T) = (1-\varepsilon)\,\Pi^{\mu\nu}_{\mathbf{V}}(q;T=0) + \varepsilon\,\Pi^{\mu\nu}_{\mathbf{A}}(q;T=0)$$

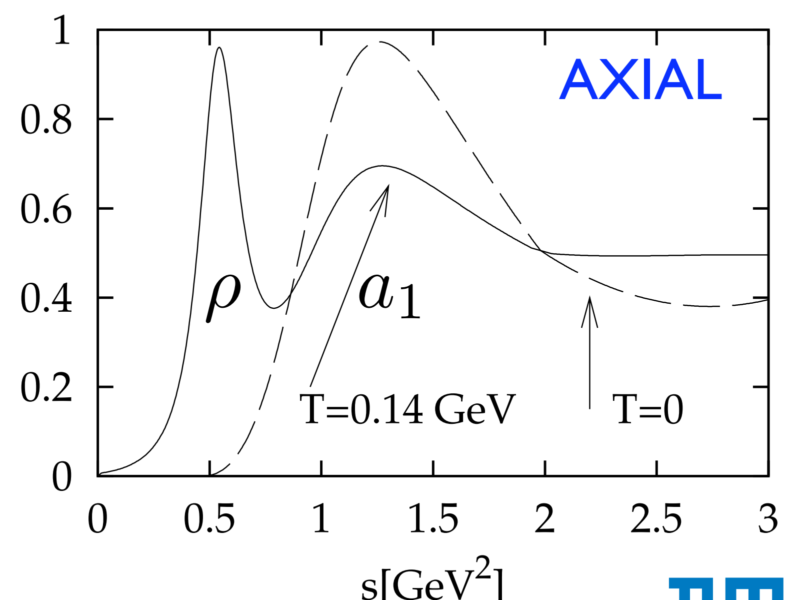
$$\Pi^{\mu\nu}_{\mathbf{A}}(q;T) = (1-\varepsilon)\,\Pi^{\mu\nu}_{\mathbf{A}}(q;T=0) + \varepsilon\,\Pi^{\mu\nu}_{\mathbf{V}}(q;T=0)$$

$$\varepsilon = \frac{\mathbf{T^2}}{\mathbf{6f_\pi^2}} + \mathcal{O}(T^4)$$

V. I. Eletsky, B.L. Ioffe: Phys. Rev. D47 (1993) 3083, D51 (1995) 2371

E. Marco, R. Hofmann, W.W. :
Phys. Lett. B 539 (2002) 88





2.
QCD SUM RULES
in
Vacuum and **In-Medium**

Reminder of **QCD SUM RULES**

(Shifman, Vainshtein, Zakharov)

- CURRENT-CURRENT CORRELATION FUNCTION:

  ... write as (twice subtracted) DISPERSION RELATION

$$\Pi(q^2) = \Pi(0) + \Pi'(0)\, q^2 + \frac{q^4}{\pi}\int ds \frac{Im\,\Pi(s)}{s^2(s - q^2 - i\epsilon)}$$

- OPERATOR PRODUCT EXPANSION (Wilson):

  ... expand at LARGE SPACELIKE $\mathbf{q^2 = -Q^2 < 0}$

$$12\pi^2\,\Pi(q^2 = -Q^2) = -c_0\, Q^2 \ln\left(\frac{Q^2}{\mu^2}\right) + c_1 + \frac{c_2}{Q^2} + \frac{c_3}{Q^4} + \ldots$$

- for **isovector vector** current:

$$c_0 = \frac{3}{2}\left(1 + \frac{\alpha_s}{\pi}\right) \quad \text{(leading)} \qquad c_1 = \frac{9}{2}\left(m_u^2 + m_d^2\right) \quad \text{(small)}$$

$$c_2 = \frac{\pi^2}{2}\langle\frac{\alpha_s}{\pi}G^2\rangle + 6\pi^2\left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right)$$

$c_3 = \ldots$ condensates of higher dimension (uncertain)

$\langle\frac{\alpha_s}{\pi}G^2\rangle$: $\mathbf{(0.33\ GeV)^4}$ (small)

$m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle$: $\mathbf{-\,m_\pi^2\, f_\pi^2 \simeq -(0.11\,GeV)^4}$

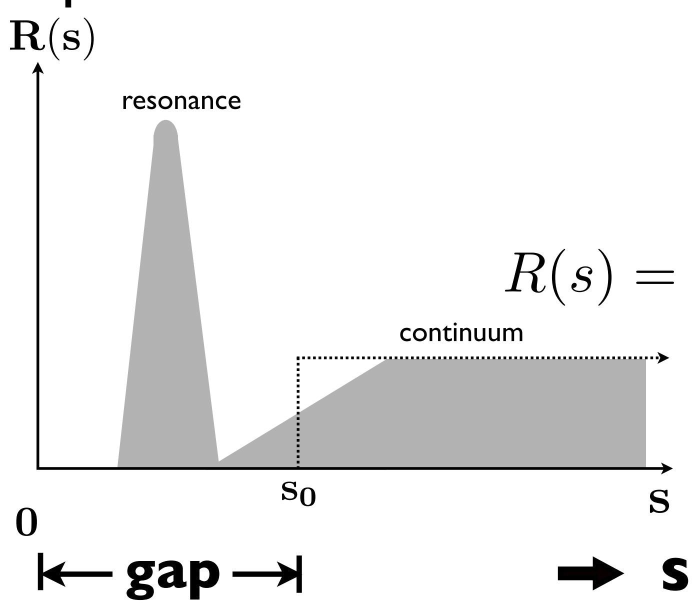
QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS

F. Klingl, W.W. : EPJ A 4 (1999) 225

- perform BOREL transformation:

$$12\pi^2\,\Pi(0) + \int_0^\infty ds\, R(s)\exp\left(-\frac{s}{\mathcal{M}^2}\right) = c_0\,\mathcal{M}^2 + c_1 + \frac{c_2}{\mathcal{M}^2} + \frac{c_3}{2\mathcal{M}^4} + \ldots$$

- separate **resonance** and **continuum** parts of spectral function:



example ρ meson:

$$R(s) = R_\rho(s)\,\theta(s_0 - s) + \frac{3}{2}\left(1 + \frac{\alpha_s}{\pi}\right)\,\theta(s - s_0)$$

take $\mathcal{M}^2 >> s_0$ and expand:

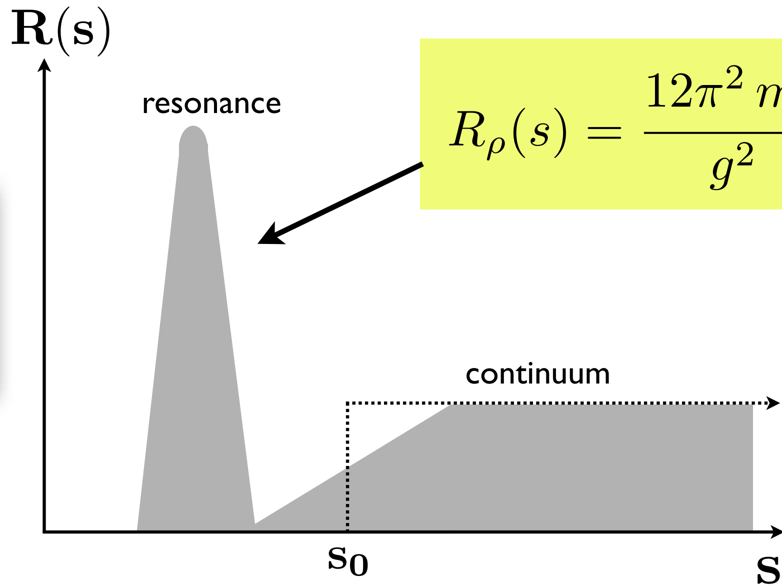
➡ **sum rules** for **MOMENTS** of R(s):

- 0th moment: $\displaystyle\int_0^{s_0} ds\, R(s) = \frac{3}{2}\, s_0\left(1 + \frac{\alpha_s(s_0)}{\pi}\right) + c_1 - 12\pi^2\,\Pi(0)$

- 1st moment: $\displaystyle\int_0^{s_0} ds\, s\, R(s) = \frac{3}{4}\, s_0^2\left(1 + \frac{\alpha_s(s_0)}{\pi}\right) - c_2$

QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS (contd.)

- relation between **gap** $\sqrt{\mathbf{s}_0}$ and **chiral** s.b. scale $4\pi\,\mathbf{f}_\pi$
- examine ρ meson in vacuum, leading terms:
- **assume**: $\sqrt{s_0} = 4\pi\, f_\pi$



$$R_\rho(s) = \frac{12\pi^2\, m_\rho^2}{g^2}\,\delta(s - m_\rho^2)$$

0th moment:

$$\int_0^{s_0} ds\, R_\rho(s) = \frac{3}{2} s_0$$

$$\rightarrow\ \mathbf{m}_\rho^2 = 2\,\mathbf{g}^2\,\mathbf{f}_\pi^2$$

1st moment:

$$\int_0^{s_0} ds\, s\, R_\rho(s) = \frac{3}{4} s_0^2$$

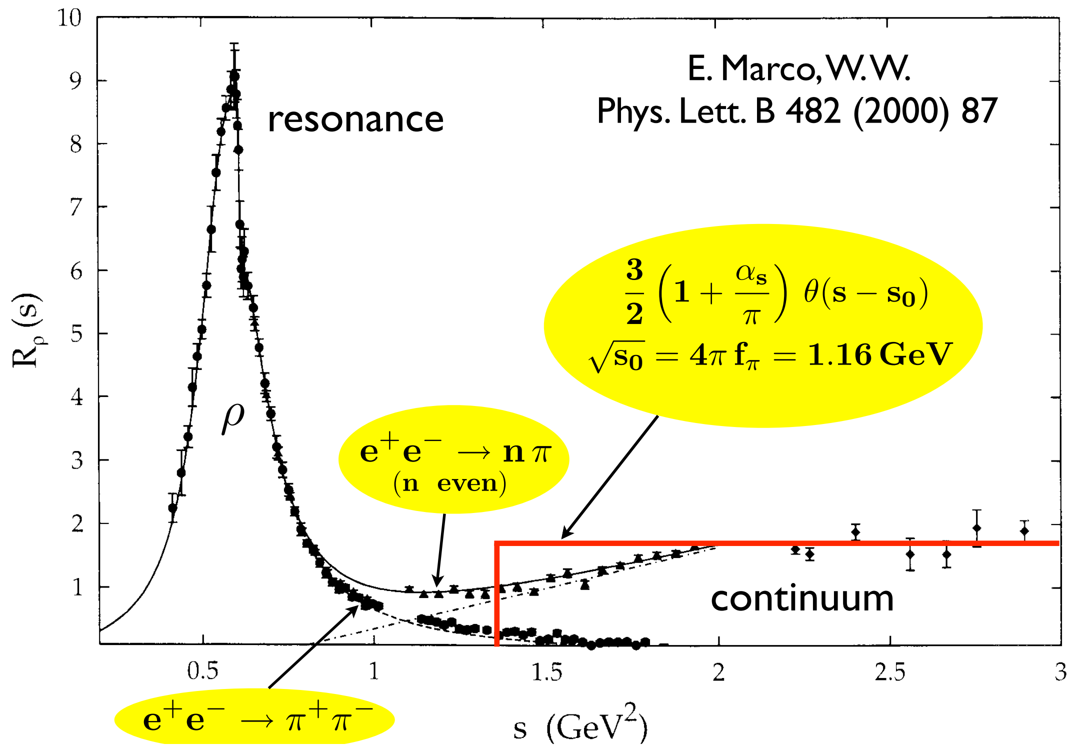
$$\rightarrow\ \mathbf{g} = 2\pi$$

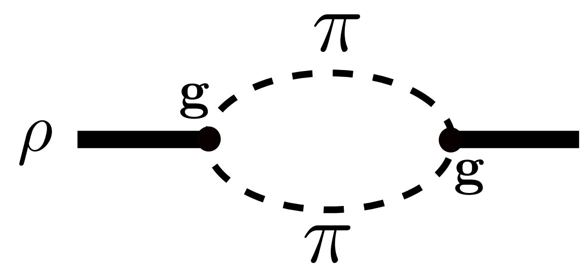
(KSFR + Weinberg + Wess-Zumino)

QCD Vacuum Sum Rules: detailed ρ meson analysis

- use: chiral effective field theory incl. vector mesons

F. Klingl, N. Kaiser, W.W.: Nucl. Phys. A 624 (1997) 527





$$\mathbf{g = 6.05}$$

$$\alpha_s(s_0) = 0.5$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.33\,\mathrm{GeV})^4$$

$$m_\pi^2 f_\pi^2 = (0.11\,\mathrm{GeV})^4$$

$$\int_0^{s_0} ds\, R(s) = \frac{3}{2} s_0 \left(1 + \frac{\alpha_s(s_0)}{\pi}\right)$$

$$\int_0^{s_0} ds\, s\, R(s) = \frac{3}{4} s_0^2 \left(1 + \frac{\alpha_s(s_0)}{\pi}\right) - c_2$$

$$c_2 = \frac{\pi^2}{2} \langle \frac{\alpha_s}{\pi} G^2 \rangle - 6\pi^2 m_\pi^2 f_\pi^2$$

with $\sqrt{s_0} = 4\pi f_\pi$

↓

consistency between 0th and 1st moments to < 2%

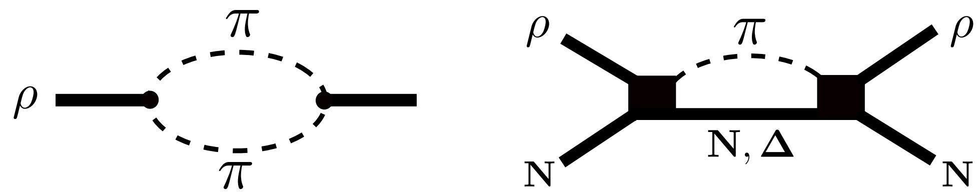
Y. Kwon, M. Procura, W.W. (2007)

In-Medium Spectral Functions of VECTOR MESONS

framework:

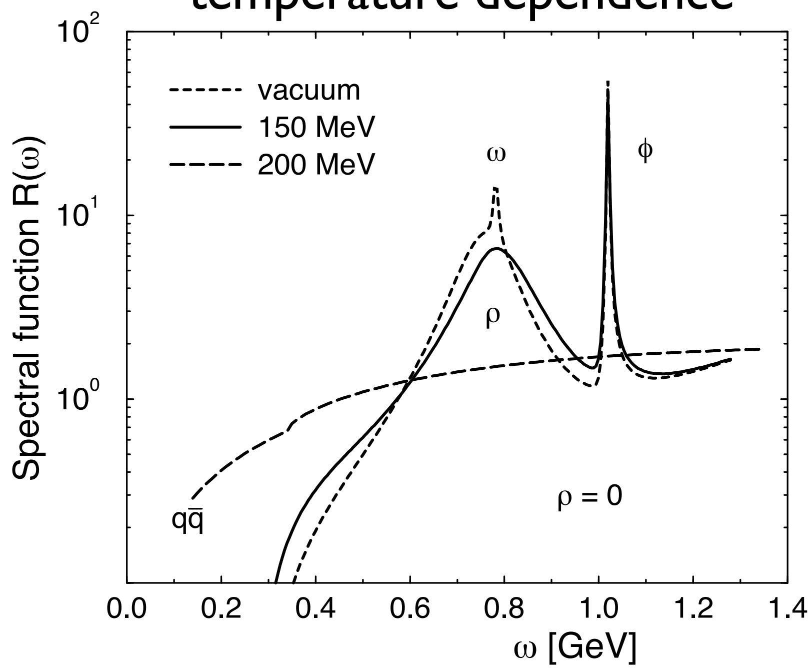
- chiral effective field theory + vector mesons + baryons

- in-medium
  ρ meson self-energy
  $(q^\mu = (\omega, \vec{q} = 0))$



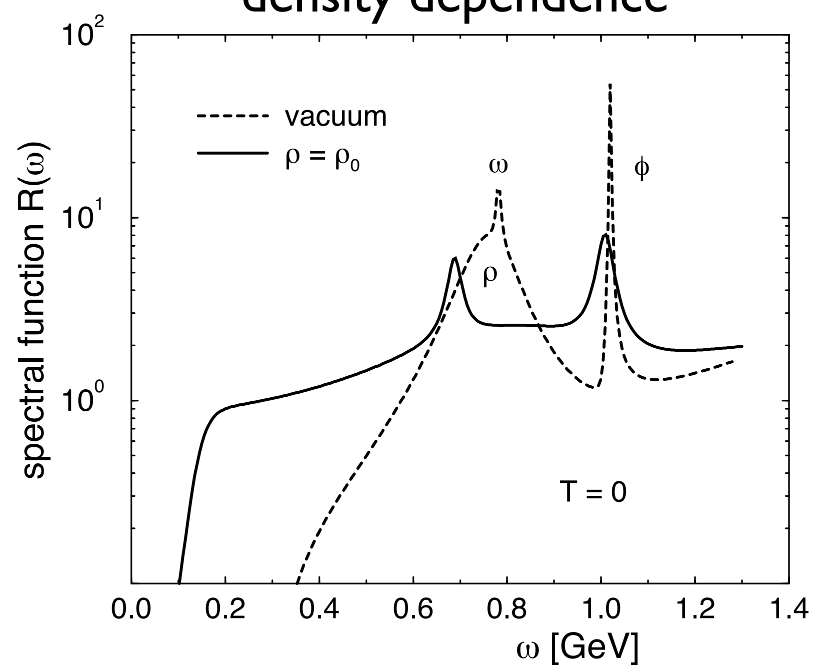
$$\mathbf{\Pi}_\rho(\omega; \rho_B, T) = \mathbf{\Pi}_\rho(\omega; \rho_B = 0, T) - \rho_B\, \mathcal{T}_{\rho N}(\omega)$$

temperature dependence



R. A. Schneider, W.W.
EPJ A9 (2000) 357

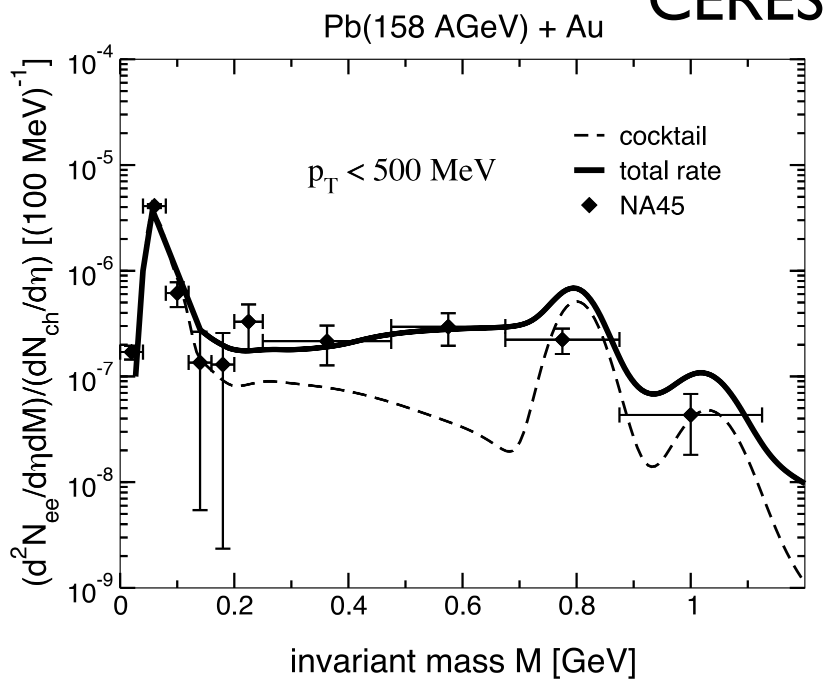
density dependence

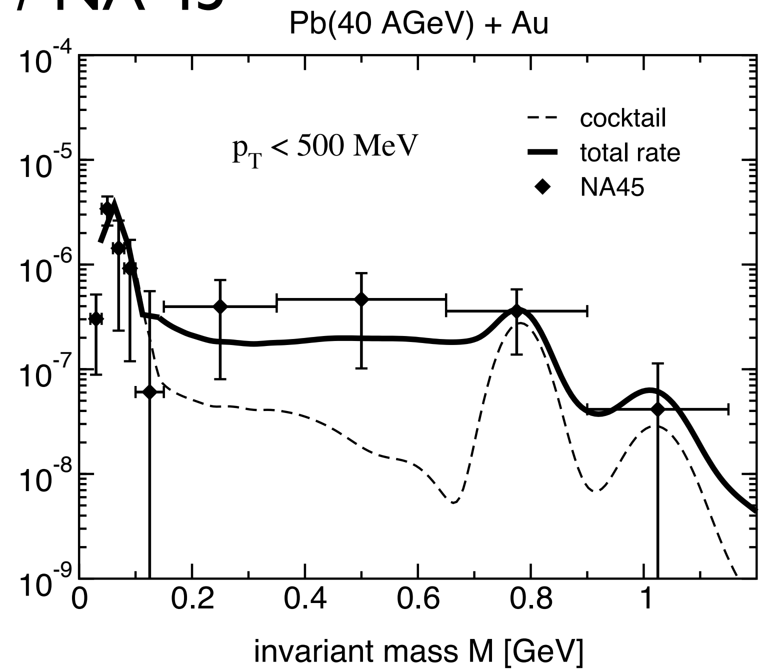


F. Klingl, N. Kaiser, W.W. :
Nucl. Phys. A 624 (1997) 527

Applications: Dileptons from HI Collisions

CERES / NA 45



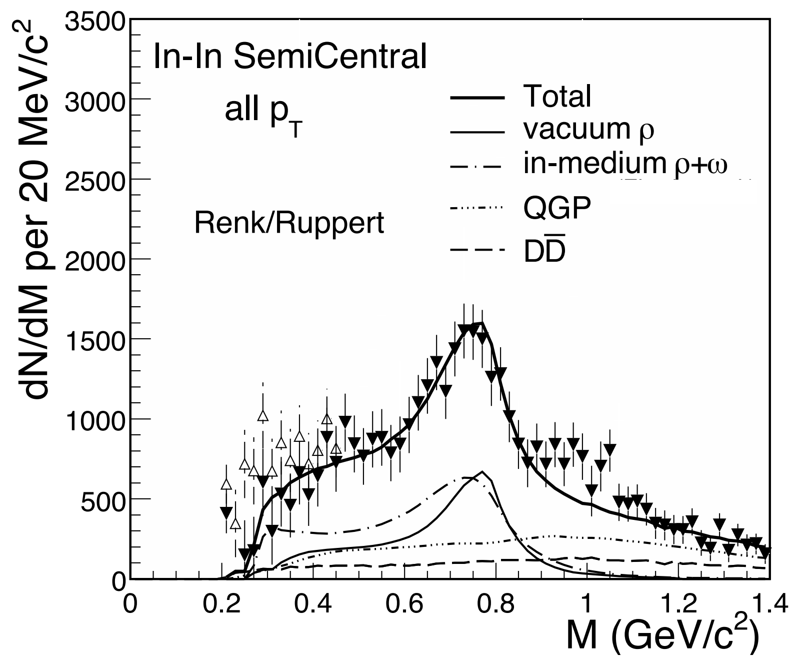


T. Renk,
R. A. Schneider,
W.W.:
PRC 66 (2002)
014902

see also:

R. Rapp,
J. Wambach,
Adv. Nucl. Phys.
25 (2000) 1

NA 60

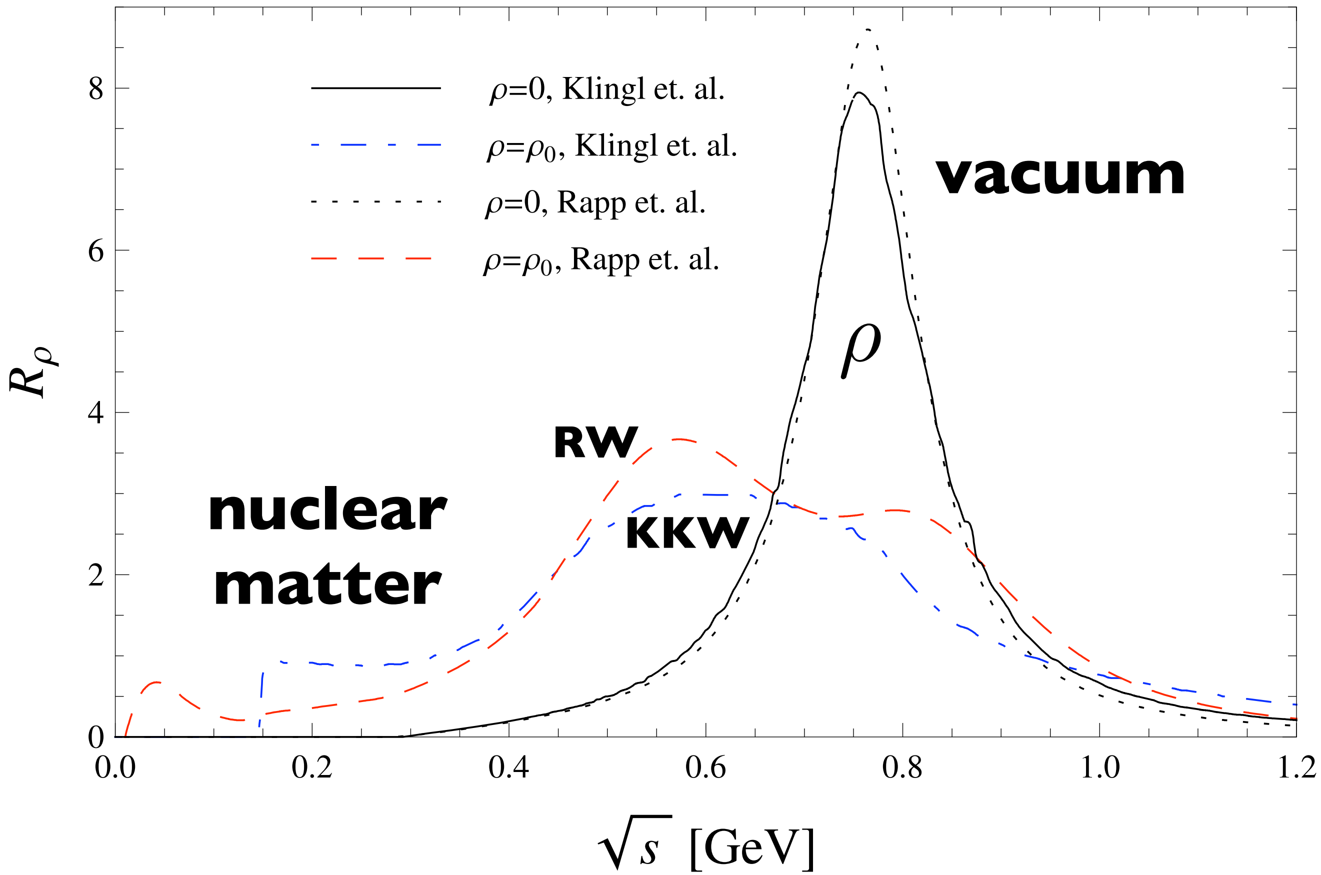


J. Ruppert, T. Renk,
EPJ C49 (2007) 219

In-Medium ρ MESON Spectral Functions

RW : R. Rapp, J. Wambach; Adv. Nucl. Phys. 25 (2000) 1

KKW : F. Klingl, N. Kaiser, W.W. ; Nucl. Phys. A 624 (1997) 527



2.4 IN-MEDIUM QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS

consider finite baryon density ρ, T = 0

$\sim 0.16\,\frac{\rho}{\rho_0}$

- expect: density dependent **gap** / **chiral** s.b. scale

$$\sqrt{s_0} = 4\pi\, f_\pi \quad \rightarrow \quad \sqrt{s_0^*(\rho)} = 4\pi\, f_\pi^*(\rho) \simeq 4\pi\, f_\pi \left(1 - \frac{\sigma_N}{2m_\pi^2\, f_\pi^2}\,\rho\right)$$

- case study: in-medium ρ meson spectral function

0th moment:
$$\int_0^{s_0^*} ds\, R(s,\rho) = \frac{3}{2}\, s_0^* \left(1 + \frac{\alpha_s(s_0^*)}{\pi}\right) - \frac{3\pi^2}{M_N}\,\rho$$

1st moment:
$$\int_0^{s_0^*} ds\, s\, R(s,\rho) = \frac{3}{4}\, {s_0^*}^2 \left(1 + \frac{\alpha_s(s_0^*)}{\pi}\right) - (c_2 + \delta c_2(\rho))$$

vacuum condensates:
$$c_2 = \frac{\pi^2}{2}\langle\frac{\alpha_s}{\pi}G^2\rangle + 6\pi^2\left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right)$$

\+

in-medium changes:
$$\delta c_2(\rho) = 3\pi^2\rho\left(M_N\, A_1 - \frac{4}{27}M_N^{(0)} + 2\sigma_N\right)$$

NO dependence on (unknown) **four-quark condensates** ...

... which enter only in the 2nd moment

IN-MEDIUM QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS (contd.)

- input (r.h.s.): $$\delta c_2(\rho) = 3\pi^2\rho\left(M_N\,A_1 - \frac{4}{27}M_N^{(0)} + 2\sigma_N\right)$$

1st moment of parton distribution from DIS

$A_1 \simeq 1$

2 x momentum fraction carried by quarks in the nucleon

density dependence of gluon condensate

$M_N^{(0)} \simeq 0.88\,GeV$

nucleon mass in the chiral limit (ChPT)

density dependence of quark condensate

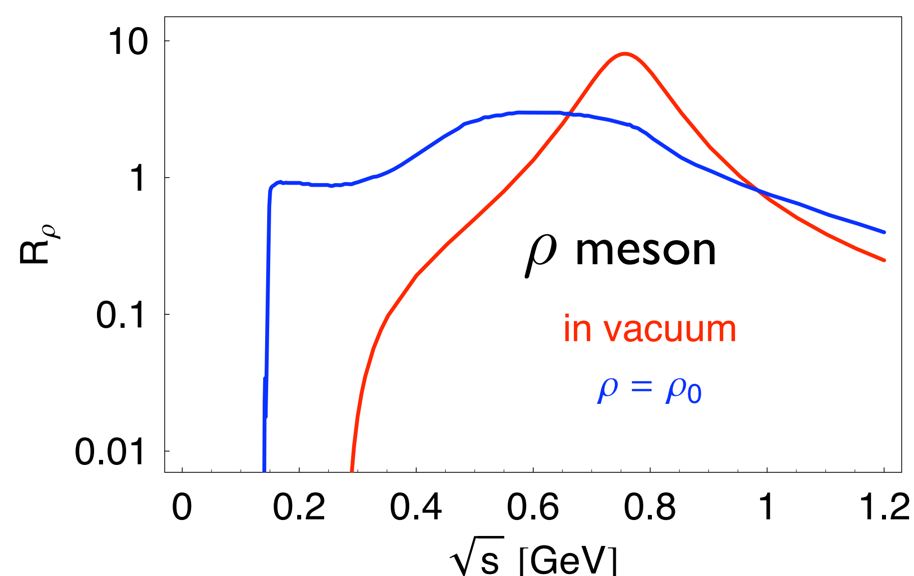
$\sigma_N \simeq 45\,MeV$

pion-nucleon sigma term (emp.)

- input (l.h.s.):

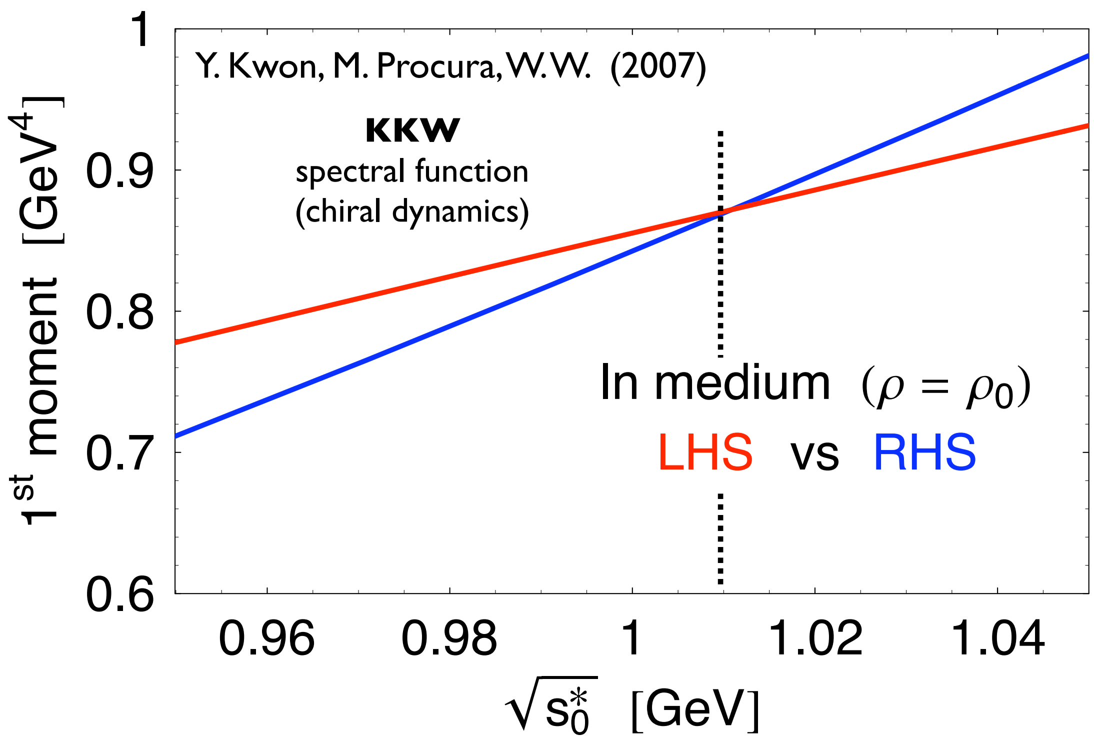
calculated in-medium ρ meson spectral function

F. Klingl, N. Kaiser, W.W.: Nucl. Phys. A 624 (1997) 527



IN-MEDIUM QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS (contd.)

- detailed analysis : LHS $\int_0^{s_0^*} ds\, s\, R(s,\rho) = \mathcal{F}(s_0^*,\rho) \int_0^{s_0^*} ds\, R(s,\rho)$ RHS



- result:

$$\sqrt{\mathbf{s_0^*}} = 4\pi\, \mathbf{f}_\pi^*(\rho = \rho_\mathbf{0}) = \mathbf{1.01\, GeV}$$

(within 3 %)

- compare vacuum:

$$\sqrt{\mathbf{s_0}} = 4\pi\, \mathbf{f}_\pi = \mathbf{1.16\, GeV}$$

- ... consistent with:

$$\frac{\mathbf{f}_\pi^*(\rho)}{\mathbf{f}_\pi} \simeq \mathbf{1} - \frac{\sigma_\mathbf{N}}{\mathbf{2 m_\pi^2 f_\pi^2}}$$

- about Brown-Rho Scaling :

define $\bar{\mathbf{m}}^2(\rho) = \dfrac{\int_0^{s_0^*} ds\, s\, R(s,\rho)}{\int_0^{s_0^*} ds\, R(s,\rho)}$, then $\dfrac{\bar{\mathbf{m}}(\rho_\mathbf{0})}{\mathbf{m}_\rho} \simeq \dfrac{\mathbf{f}_\pi^*(\rho_\mathbf{0})}{\mathbf{f}_\pi}$ within < 3%

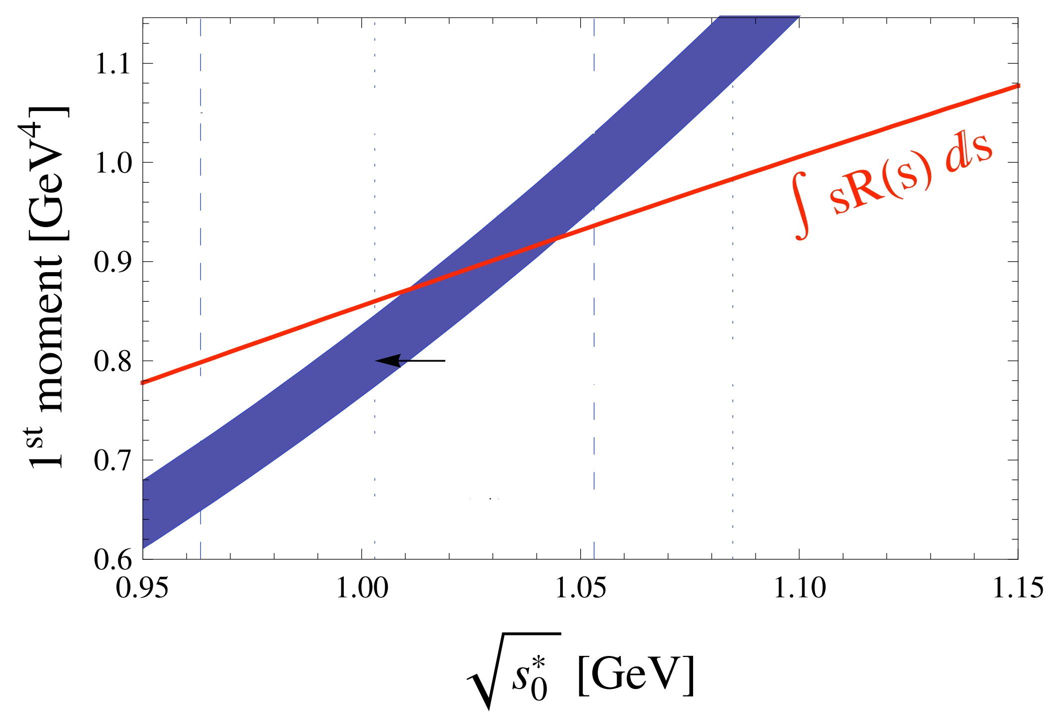
IN-MEDIUM QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS (contd.)

- detailed error analysis
- largest uncertainty from perturbative QCD correction

$$\int_0^{s_0^*} ds\, R(s,\rho) = \frac{3}{2}\, s_0^* \left(1 + \frac{\alpha_s(s_0^*)}{\pi}\right) - \frac{3\pi^2}{M_N}\,\rho \qquad \int_0^{s_0^*} ds\, s\, R(s,\rho) = \frac{3}{4}\, s_0^{*2} \left(1 + \frac{\alpha_s(s_0^*)}{\pi}\right) - (c_2 + \delta c_2(\rho))$$

$$c_2 = \frac{\pi^2}{2}\langle\frac{\alpha_s}{\pi}G^2\rangle + 6\pi^2\left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right) \qquad \delta c_2(\rho) = 3\pi^2\rho\left(M_N\, A_1 - \frac{4}{27}M_N^{(0)} + 2\sigma_N\right)$$

- error band assuming $\mathbf{0.4 \le \alpha_s(s_0^*) \le 0.7}$



KKW
spectral function
(chiral dynamics):

BR scaling !

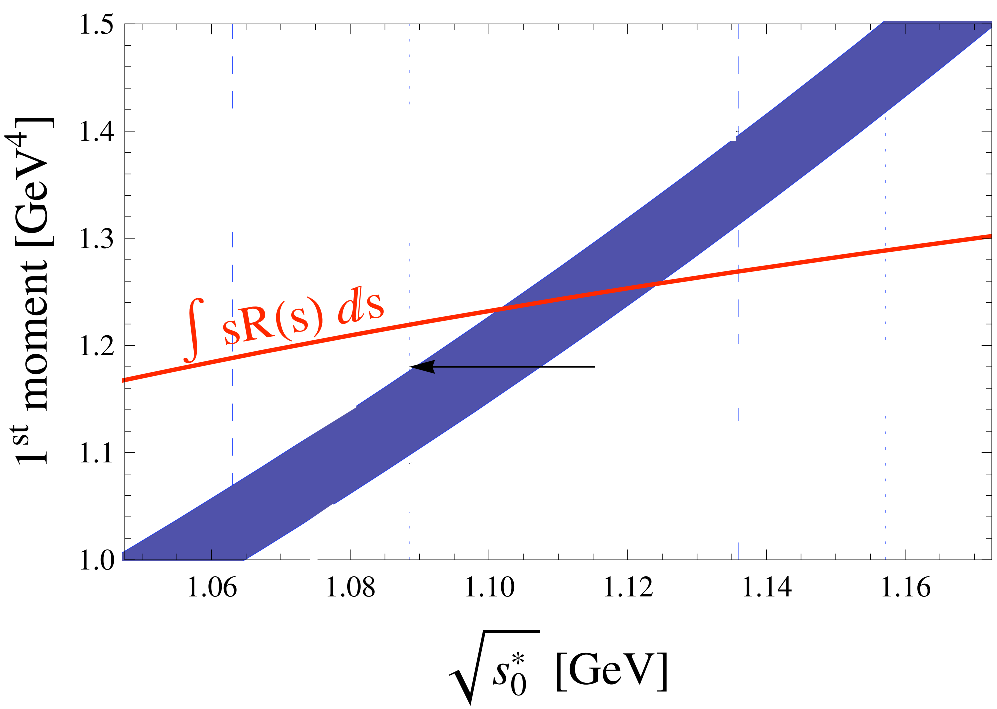
IN-MEDIUM QCD SUM RULES for MOMENTS of SPECTRAL FUNCTIONS (contd.)

- detailed error analysis
- largest uncertainty from perturbative QCD correction

$$\int_0^{s_0^*} ds\, R(s,\rho) = \frac{3}{2}\, s_0^* \left(1 + \frac{\alpha_s(s_0^*)}{\pi}\right) - \frac{3\pi^2}{M_N}\,\rho \qquad \int_0^{s_0^*} ds\, s\, R(s,\rho) = \frac{3}{4}\, s_0^{*2} \left(1 + \frac{\alpha_s(s_0^*)}{\pi}\right) - (c_2 + \delta c_2(\rho))$$

$$c_2 = \frac{\pi^2}{2}\langle\frac{\alpha_s}{\pi}G^2\rangle + 6\pi^2\left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right) \qquad \delta c_2(\rho) = 3\pi^2\rho\left(M_N\, A_1 - \frac{4}{27}M_N^{(0)} + 2\sigma_N\right)$$

- error band assuming $\mathbf{0.4 \le \alpha_s(s_0^*) \le 0.7}$



RW spectral function (resonance dynamics):

BR scaling ?

3. CONCLUSIONS

QCD SUM RULES for
first two **MOMENTS** of **SPECTRAL DISTRIBUTION**
are **accurate**, both in **vacuum** and **in-medium**

“**Mass Shift**“ against “**Broadening**“ is **not** an issue:
investigate **1st moment of spectral function**

Brown-Rho Scaling
is seen as a statement about the first moment of the spectrum:

$$\bar{\mathbf{m}}^2(\rho) = \frac{\int_0^{s_0^*} ds\, s\, R(s,\rho)}{\int_0^{s_0^*} ds\, R(s,\rho)} \qquad \frac{\bar{\mathbf{m}}(\rho_\mathbf{0})}{\mathbf{m}_\rho} \simeq \frac{\mathbf{f}_\pi^*(\rho_\mathbf{0})}{\mathbf{f}_\pi}$$