IN-MEDIUM $\phi$ MESON SUM RULES and STRANGENESS CONTENT of the NUCLEON

- The $\phi(1020)$ in vacuum and in nuclear matter
- QCD sum rules and the $\phi(1020)$
- $\phi$ spectral moments and QCD condensates
- Strangeness sigma term of the nucleon
- Contact with reality: J-PARC experiment E16

PREAMBLE & PART I

Brief History of Dilepton Production
and
In-medium Properties of Vector Mesons

- Vector mesons $\rho, \omega, \phi, J/\psi$: basic dipole modes of QCD

- Vector meson spectral functions: $V \rightarrow e^+e^-, \mu^+\mu^-$
Chiral symmetry breaking

SPONTANEOUS

Nambu - Goldstone

SU(3)$_L \times$ SU(3)$_R$

EFFECTIVE FIELD THEORY

SU(2)$^*$ GeV

\[
\begin{align*}
m_u &= 2.3 \pm 0.7 \text{ MeV} \\
m_u/m_d &\sim 0.4 - 0.6 \\
m_s &= 95 \pm 5 \text{ MeV} \\
&\quad (\mu \approx 2 \text{ GeV})
\end{align*}
\]

\[
\begin{align*}
m_c &= 1.28 \pm 0.03 \text{ GeV} \\
m_b &= 4.18 \pm 0.03 \text{ GeV} \\
m_t &= 173.2 \pm 1.2 \text{ GeV}
\end{align*}
\]
Spontaneous CHIRAL SYMMETRY Breaking and Current Algebra / Sum Rule Relations

\[ \frac{m_{a_1}}{m_\rho} \simeq 1.6 \]

\[ m_\rho = 775.3 \pm 0.3 \text{ MeV} \quad \Gamma_\rho \simeq 150 \text{ MeV} \]

\[ m_{a_1} = 1230 \pm 40 \text{ MeV} \quad \Gamma_{a_1} \sim 400 \text{ MeV} \]

- Characteristic scale of spontaneous chiral symmetry breaking:
  \[ \Lambda_{\text{chiral}} \sim 4\pi f_\pi \simeq 1.2 \text{ GeV} \]
  with \[ f_\pi \simeq 92.2 \text{ MeV} \] (pion decay constant)

- Current Algebra
  Weinberg Sum Rules:
  \[ m_{a_1}^2 - m_\rho^2 = 8\pi^2 f_\pi^2 \]
  \[ m_{a_1} = \sqrt{2} m_\rho = 4\pi f_\pi \]

Starting point: correlation tensor of e.m. current $j_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle T[j_\mu(x) j_\nu(0)] \rangle$$

(in-medium: density and temperature dependent)

Spectral function: $R(s) = \frac{4\pi}{s} \text{Im} \Pi_\mu^\mu(s = q^2)$

(in vacuum: $R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$)
QCD SUM RULES for VECTOR MESONS

- **Energy-weighted sum rules**
  Model-independent QCD sum rules for moments of the spectral function $R(s)$ in the chiral limit ($m_q \rightarrow 0$)

\[
\int_0^{\infty} ds \ [R(s) - R_{pQCD}(s)] = 0
\]

\[
\int_0^{\infty} ds \ s [R(s) - R_{pQCD}(s)] = C_4 \langle G G \rangle \quad \text{(dim. 4 gluon condensate)}
\]

\[
\int_0^{\infty} ds \ s^2 [R(s) - R_{pQCD}(s)] = C_6 [\langle \bar{q}_i \Gamma q_i \bar{q}_j \Gamma q_j \rangle] \quad \text{(dim. 6 four-quark condensates)}
\]

- Additional terms on right-hand sides when $m_q \neq 0$

- **In-medium sum rules:**
  QCD condensates become functions of density and temperature

- **Sum rule analysis** of in-medium $\rho$ meson and its coupling to $\pi \pi$ continuum:
  broadening and shift of 1st moment

SPECTRAL FUNCTIONS of VECTOR MESONS

in vacuum and in-medium: typical examples


\[ R^{(I=1)}(s) \]

\[ \rho \]

\[ \phi \]

\[ \sqrt{s} \text{ [GeV]} \]

\[ 10^{-2} \quad 10^{-1} \quad 10^{0} \quad 10^{1} \quad 10^{2} \]

\[ \begin{aligned}
&\ldots \rho=0 \\
&\ldots \rho=\rho_0/2 \\
&\ldots \rho=\rho_0
\end{aligned} \]

\[ \sqrt{s} \text{ [GeV]} \]

\[ 0.60 \quad 0.70 \quad 0.80 \quad 0.90 \quad 1.00 \quad 1.10 \quad 1.20 \quad 1.30 \]

\[ \begin{aligned}
&\ldots \rho=0 \\
&\ldots \rho=\rho_0/2 \\
&\ldots \rho=\rho_0
\end{aligned} \]
In-medium properties of vector mesons

Dileptons from relativistic heavy-ion collisions

T. Renk, R.A. Schneider, W.W.

R. Rapp, J. Wambach

SPS conditions
CERES/NA45

hadron
phase

quark-gluon
phase

in-medium \( \rho \) meson and coupling to \( \pi \pi \) continuum:
- broadening and shift of 1st moment
In-medium properties of vector mesons

Dileptons from relativistic heavy-ion collisions

NA 60 data vs. theory

PHENIX @ RHIC

- large contribution from in-medium rho spectral function

H. van Hees, R. Rapp

PHENIX low-mass spectrum

Afanasiev et al. (2007)
**PART II**

\( \phi \) **Meson in Vacuum** and in a **Nuclear Medium**

- **Vacuum properties**
  \[ m_\phi = 1019.44 \pm 0.02 \text{ MeV} \quad \Gamma = 4.27 \pm 0.03 \text{ MeV} \]

- **Decay modes:**
  \( K^+K^- (50\%) \), \( K^0_LK^0_S (35\%) \), \( 3\pi (15\%) \)
Using Eq. (21) instead of Eq. (17), the finite energy sum rules of Eqs. (18-20) are also shown [25, 26]. A fit to the data is presented together with a (linear) ramp function (dotted).

The hyperon intermediate states for two energies (1.0 GeV for the meson (taken at rest), as a function of the squared invariant mass at rest), as a function of the squared invariant mass $s$.

The right-hand side of Eq. (18) as functions of the delineation scale $l$ and the parameter $W_{\pi}$. It is evident that the spectral function (17) and the corresponding plus earlier $e^+e^- \rightarrow K^+K^- + n\pi$ data

Figure 3: The left-hand and right-hand sides of Eqs. (18) and (19) as functions of the delineation scale $l$ and the parameter $W_{\pi}$. It is evident that the spectral function (17) and the corresponding...
\[ \phi \text{ MESON SPECTRAL FUNCTION} \]

in NUCLEAR MATTER

\[ \phi \text{ from Chiral SU(3) EFT} \]

F. Klingl, T. Waas, W. W.

\[ \sqrt{s} \text{ [GeV]} \]

- Asymmetric broadening of spectrum
- Spectral strength at lower masses
IN-MEDIUM $\phi$ MESON SPECTRAL FUNCTIONS and SUM RULES

- QCD sum rules analysis
- Implications for strangeness sigma term of the nucleon

Starting point: strange quark vector current $j^\mu(x) = -\frac{1}{3} \bar{s}(x) \gamma^\mu s(x)$

and its correlator $\Pi(q)^{\mu\nu} = i \int d^4x \ e^{iqx} \langle T[j^\mu(x)j^\nu(0)] \rangle\rho$

Spectral function $R(q) \propto \frac{Im \Pi^\mu_\mu(q)}{q^2}$

- Operator product expansion of QCD
- Finite energy sum rules for moments of the spectral function
Borel transform: \[
\frac{1}{M^2} \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 + \frac{c_2}{M^2} + \frac{c_4}{M^4} + \frac{c_6}{2M^6} + \ldots
\]

\[
c_0 = \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{\pi}\right)
\]

\[
c_2 = -\frac{3m_s^2}{2\pi^2}
\]

\[
c_4 = \frac{1}{12} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + 2m_s \left\langle \bar{s}s \right\rangle
\]

- \(\alpha_s(2 \text{ GeV}) = 0.31 \pm 0.01\) [28]
- \(m_s(2 \text{ GeV}) = 95 \pm 5 \text{ MeV}\) [28]
- \(\left\langle \bar{s}s \right\rangle(2 \text{ GeV}) = (-290 \pm 15 \text{ MeV})^3\) [29]
- \(\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.012 \pm 0.004 \text{ GeV}^4\) [30]

- Borel scale \(M\) is arbitrary
- Choose \(M^2 \gg s_0\) and expand …
**FINITE ENERGY QCD SUM RULES**

for moments of the $\phi$ spectral function

- Sum rule for 0th moment:
  $$\int_0^{s_0} ds \ R(s) = \frac{s_0}{4\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) - \frac{3m_s^2}{2\pi^2}$$

- Sum rule for 1st moment:
  $$\int_0^{s_0} ds \ s \ R(s) = \frac{s_0^2}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) - c_4$$

\[ c_4 = \frac{1}{12} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + 2m_s \langle \bar{s}s \rangle \quad \text{... in vacuum} \]

- Delineation scale between hadronic sector and perturbative QCD:

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**Figure 2:** The spectral function $\phi$ above

**Figure 3:** The left- and right-hand sides of Eqs. (18) and (19) as functions of $s_0$.

**Sum rule for 0th moment:**
$$s_0 \ c_0 + c_2$$

**Sum rule for 1st moment:**
$$\int ds \ R(s)$$

**Delineation scale between hadronic sector and perturbative QCD:**
$$\langle \left\langle G^2 \right\rangle \rangle = 1.55$$

**Gluon condensate**

**Strange quark condensate**
1st spectral moment involves in-medium gluon and strange quark condensates:

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle_\rho \simeq \langle \frac{\alpha_s}{\pi} G^2 \rangle + \langle N \mid \frac{\alpha_s}{\pi} G^2 \mid N \rangle \rho = \langle \frac{\alpha_s}{\pi} G^2 \rangle - \frac{8}{9} (M_N - \sigma_{\pi N} - \sigma_{SN}) \rho
\]

\[
\langle \bar{s}s \rangle_\rho \simeq \langle \bar{s}s \rangle + \langle N \mid \bar{s}s \mid N \rangle \rho = \langle \bar{s}s \rangle + \frac{\sigma_{SN}}{m_s} \rho
\]

... plus correction from twist-2 operator proportional to:

\[
A_2^s = 2 \int_0^1 dx \ x [s(x) + \bar{s}(x)] = (4.4 \pm 1.1) \cdot 10^{-2}
\]

**Sigma terms of the nucleon:**

\[
\sigma_{\pi N} = 2m_q \langle N \mid \bar{q}q \mid N \rangle \simeq 50 \text{ MeV}
\]

\[
\sigma_{SN} = m_s \langle N \mid \bar{s}s \mid N \rangle \text{ ... large uncertainties}
\]
SUM RULE for $\phi$ SPECTRAL FUNCTION in the NUCLEAR MEDIUM

- In-medium sum rule for 1st spectral moment:

$$\Delta(\rho) = \int_0^{s_0} ds \, s \, R(s; \rho) - \frac{c_0}{2} s_0^2 = -(c_4 + \delta c_4(\rho))$$

$$\delta c_4(\rho) = \left[ \left( A_2^s - \frac{2}{27} \right) M_N + \frac{2}{27} \left( 28 \sigma_{sN} + \sigma_{\pi N} \right) \right] \rho$$

strong sensitivity to strangeness sigma term

- In-medium change of 1st spectral moment:

$$\frac{\Delta(\rho) - \Delta(\rho = 0)}{\Delta(\rho = 0)} = \frac{\delta c_4(\rho)}{c_4}$$

( for narrow resonance: interpretation as mass shift

$$\frac{\Delta(\rho)}{\Delta(\rho = 0)} - 1 \simeq \frac{2 \delta m_\phi(\rho)}{m_\phi}$$

PHYSIK DEPARTMENT
PART III

In-medium $\phi$ Meson Spectral Functions and Strange-Quark Content of the Nucleon
SUM RULE for $\phi$ SPECTRAL FUNCTION in the NUCLEAR MEDIUM and STRANGENESS SIGMA TERM of the nucleon

- **In-medium shift of 1st moment of $\phi$ spectral function**

$$\frac{\Delta(\rho) - \Delta(\rho = 0)}{\Delta(\rho = 0)} = \frac{\delta c_4(\rho)}{c_4} = \left[ a + b \left( \frac{\sigma_{sN}}{M_N} \right) \right] \frac{\rho}{\rho_0}$$

- **Default case:** NO shift implies

$$\sigma_{sN} \simeq (12.5 \pm 2.2) \text{ MeV}$$

$$a \simeq 0.01$$

$$b = -0.75 \pm 0.09$$

**zero-width limit:**

Ph. Gubler, K. Ohtani
Correlation between in-medium $\phi$ “mass” and strangeness content of the nucleon

\[ \frac{2 \delta m_\phi (\rho = \rho_0)}{m_\phi} \]

\begin{align*}
\sigma_{sN} [\text{MeV}] & \quad 0 \quad 25 \quad 50 \quad 75 \quad 100 \quad 125 \quad 150 \quad 175 \quad 200 \\
[\Delta(\rho_0) - \Delta(0)] / \Delta(0) & \quad -0.16 \quad -0.14 \quad -0.12 \quad -0.1 \quad -0.08 \quad -0.06 \quad -0.04 \quad -0.02 \quad 0
\end{align*}

- **Scalar strange-quark density of the nucleon**

\[ y_s = \frac{\langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle} = \left( \frac{\bar{m}_q}{m_s} \right) \frac{\sigma_{sN}}{\sigma_{\pi N}} \]

- **example:** $y_s \simeq 0.04$ for $\sigma_{sN} \simeq \sigma_{\pi N} \simeq 50 \text{ MeV}$ with $\frac{\bar{m}_q}{m_s} \simeq 0.04$
STRANGENESS SIGMA TERM of the NUCLEON from LATTICE QCD

Comparison to previous results

FIG. 14. A comparison of the result of our calculation of the strangeness $\sigma_s$ term $\sigma_s = m_s \langle N|\bar{s}s|N \rangle$ with those of other groups. The statistical errors are denoted by black error bars and the total errors are denoted by blue error bars.

Recent:

- Y.B. Yang et al., PR D94 (2016) 054503
- S. Dürr et al., PRL 116 (2016) 172001
- A. Abdel-Rehim et al., PRL 116 (2016) 252001
OUTLOOKS

Contact with Reality:
E325 (KEK) and E16 (J-PARC)
Hints about in-medium effects on the $\phi$ spectral function

K. Muto et al.

Simple Breit-Wigner fit of excess spectrum below resonance:

$\sim 35$ MeV downward mass shift at $\rho = \rho_0$

Looking forward to J-PARC E16
Expected results of J-PARC E16

KEK E325 results

Fraction of excess bg = p/M of f

Double peak

System size dep.

J-PARC E16 expected results

\( \beta \gamma < 0.5 \)

Dispersion relation

\( \phi(1020) \)

Blue: expected w/o spectral change

\( \chi^2/\text{ndf} = 83/50 \)

\( \phi(1020) \)

Pb

~35MeV

Measured by E325

\( \Delta M \approx 35 \text{MeV} \)

Expected

K. Aoki    J-PARC    PAC    12 Jan 2017
QCD sum rule analysis implies a well defined hierarchy of sum rules for $n$th moments

$$\int ds \, s^n R(s)$$

of vector meson spectral functions and their relation to QCD condensates of corresponding dimensions.

- Meson spectral functions in vacuum and in nuclear medium:
  - In-medium sum rule for 1st spectral moment ($n = 1$) \( \rightarrow \) strangeness content of the nucleon.

- Sum rules for higher moments:
  - Vacuum and in-medium sum rule for 2nd spectral moment ($n = 2$) \( \rightarrow \) constraints for 4-quark condensates.

Experimental access through accurate dilepton measurements
  - KEK E325 (medium effect seen, but low statistics)
  - J-PARC E16 (forthcoming, 100 times better statistics)